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PREFACE TO THE DEVELOPMENT OF RAPID RESPONSE SYSTEMS TO DEVIATIONS OF ELECTRICITY QUALITY INDICATORS FROM STANDARD VALUES IN INTERNAL MINE ELECTRIC NETWORKS

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MONOGRAPH

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B 51 **Sinchuk I.O., Kotyakova M.G.** Preface to the development of rapid response systems to deviations of electricity quality indicators from standard values in internal mine electric networks. Edited by Professor Sinchuk O.M. Monograph – Warsaw: iScience Sp. z.o.o. – 2025 – 85 p.

The monograph presents the state of the electric power industry of mines with underground methods of mining iron ore of Ukraine. It is confirmed that the indicators of electricity quality in internal mine power supply systems do not meet the levels regulated by the relevant state standards. It is proved that it is possible to maintain electricity quality indicators with their stochastic nature at the level of standard values with sufficient positive by controllability of this process. To develop the structure of the control algorithm as a starting position, a mathematical research model has been proposed, on the basis of which it is advisable to form an appropriate preventive management structure of control systems.

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CONTENTS

LIST OF CONVENTIONAL ABBREVIATIONS

which were used in the presentation of the text of the monograph

IO	- iron ore;
PSS	- power supply system;
NCSREU	- National Commission for State Regulation of Energy and Utilities;
CDS	- Code of Distribution Systems;
SSU	- State Standard of Ukraine;
EQI	- electricity quality indicators;
RW	- research work;
EC	- electronic computer;
BLEC	- basic levels of electricity consumption;
EEI	- electricity efficiency indicators;
GDP	- gross domestic product;
E	- efficiency;
EELP	- energy-efficient level of production.

INTRODUCTION

Mining and metallurgical industry is the basic branch of the Ukrainian economy and, at the same time, the largest consumer of electric energy among other industrial enterprises of the state [1-3].

Enterprises of the above industry traditionally belong to the category of energy-intensive and energy-dependent industries, which is due to the technology of their functioning [4-8].

Moreover, according to the same technology, the components of the above-mentioned type of industrial complex - mining enterprises, including mines with underground IO mining methods as the basic components of the metallurgical production process, over time their functioning creatively degrade the energy efficiency indicator. To a large extent, this is due to the constant growth of the depths of mining - iron ore and the corresponding reaction to this - a change in the technological and technical parameters of the functioning of the electric power complex: electricity supply - power consumption of these enterprises [9-11].

It is this process that largely eliminates the positive local achievements achieved by enterprises in the direction of improving the energy efficiency of IO production. At the same time, and that it is extremely considered negative, in combination with a number of others, affects the indicators of the quality of electricity in the internal PSS of these types of enterprises [12-14].

The quality of products, including one of its most common and popular type - electricity, is a set of properties and a measure of the usefulness of products. Now, from the point of view of the present, this is already a strategically macroeconomic category.

The fact of the stability of the negative in the indicators of the quality of electric energy in the electrical networks of mining and metallurgical enterprises was formed already in the 70s of the last century [15]. At the same time, this negative related to all components of the EQI complex. However, since the 80s of the last century, the cohort of indicators has been distinguished by its level such as the deviation of the supply voltage (ΔU) and the scope of its deviations (δU) among electricity consumers. The level of non-compliance of these indicators with standard values in the conditions of PSS of mining enterprises dominated and dominates in the present among other types of industrial enterprises [15].

This unseemly situation is dramatized in the current period of time, as the EQI components in mining enterprises continue to deteriorate, which is a consequence of the timely failure to take appropriate measures to minimize the deterioration of these indicators.

Indicator of the present, in the consequences of negative changes in the technology of PSS functioning - non-compliance of existing EQI with standard values [16-18].

As an addition, we note that about 20-25% [19] of electricity is lost when it is transported through the internal PSS of mining enterprises with underground IO mining methods, and the primary levels of supply voltages of electricity receivers for the most part do not correspond to the values regulated by the corresponding SSU [10].

If we add to this the specific property of mining enterprises - a constant decrease in levels (depths) of IO production and logical support of this process - the expansion of the structural configuration of the PSS in the event of an increase in the length of the PL, the number of substations and distribution points, and, in the final version, an increase in the level of power loss and supply voltage in the segments of the analyzed electric power systems, in accordance with these changes in its negative, the EQI also increases.

If we add to this the specific property of mining enterprises - a natural and constant decrease in levels (depths) of IO production and logical support of this process - the expansion of the structural configuration of the PSS in the event of an increase in the length of the PL, the number of substations and distribution points, and, in the final version, an increase in the level of total electricity loss in the analyzed electric power systems in accordance with these changes in its negative, the EQI also increases.

However, it is logical and understandable that such prospects in the negative "slope" of both individual and generalized indicators of the quality of electricity, the issue which is absolutely necessary in the process of improving the energy efficiency of internal power grids for the energy economy of enterprises, are transformed into a global problem, without the solution of which the economy of mining enterprises, as in the industry as a whole, in the nearest Time period will come to the limit of its drama.

At the same time, in the analyzed direction - the problems of the quality of electricity in the conditions of IO production enterprises in fact today, more worthy attention from scientists and operators is needed.

One of the steps towards the "road map" of the implementation of the problem of increasing the electric power efficiency of the extraction of diesel fuel, or rather the "marker" of the level of reach of this indicator - the stabilization of electricity quality indicators is recommended by the authors for the general public of readers this monograph.

SECTION 1. ELECTRIC POWER OF UNDERGROUND MINES WITH IRON ORE MINING IN UKRAINE. STATUS, PROSPECTS

1.1 Electricity and mining and metallurgical industry: communication, problems

The power system of Ukraine by 2024 was one of the most powerful power associations among the countries of Europe. The total installed capacity of the state's power plants was 54.5 million kW. At the same time, the Dnipropetrovsk region dominates among other regions in terms of installed capacity and electricity consumption levels - more than 25% of the total volume of use [16, 17]. Along with this indicator, we note that Ukraine belongs to the cohort of countries in the world with the highest level of energy intensity of GDP, which is not a positive indicator of both the macro and microeconomics of the state. The state's industry consumes more than 70% of the national level annually. To a large extent, this is a consequence of the fact that a significant number of enterprises operated and operate in the state today, which are characterized by significant levels of energy intensity [17]. Among the largest consumers of electricity, processing industry enterprises traditionally dominate (42%), of which mining and metallurgical production accounts for more than 27% [17]. These include enterprises engaged in the extraction and processing of iron ore, a strategic type of product for the state [18, 19].

Iron ore mining on an industrial scale in Ukraine, according to available official information, has been carried out since the 1980s [20].

The ancestor of this process is the Kryvyi Rih iron ore basin with a center based in the city of Kryvyi Rih, Dnipropetrovsk region.

By the end of the 1980s, the volume of iron ore mining in the region amounted to about 300 thousand tons [21].

Later, the cohort of iron ore miners in the country increased by involving in this process such regions as Kerch (Crimea), Horishni Plavni (Poltava region), Dniprorudny (Zaporizhzhya region). However, the Kryvyi Rih iron ore basin was and continues to be the basic iron ore region of Ukraine, in which today more than 85% of the national volume of IO is mined and where about 80% of the country's pig iron is smelted in the process of further processing [22]. This reflects the position of the region as the most energy-intensive in Ukraine, which is determined, first of all, by the total reproduction of similar characteristics of the enterprises of the Kryvyi Rih iron ore basin. The system-forming and at the same time constantly growing dominant component in the total amount of energy consumed by mining enterprises is the electric power industry, which is determined by such indicators as energy intensity and electric energy efficiency of IO mining [23].

There are a number of reasons to explain this fact. The first of them is that all, without exception, domestic mines with underground mining methods IO were designed based on the technical conditions and technical task for fixed volumes of mining and the limits of underground work, which were limited to a value of up to 1000 m. exclusively. As a fact of the present - this limit has been crossed and enterprises operate at an additional up to 1000 m after the design formats, at underground horizons of 1300-1500 m, with the prospect of further reduction in depth. It is logical that such a situation provokes a corresponding increase in the length of internal PSS, the number of substations, distribution points and individual consumers of electricity in the structure of the general power supply system of the enterprise.

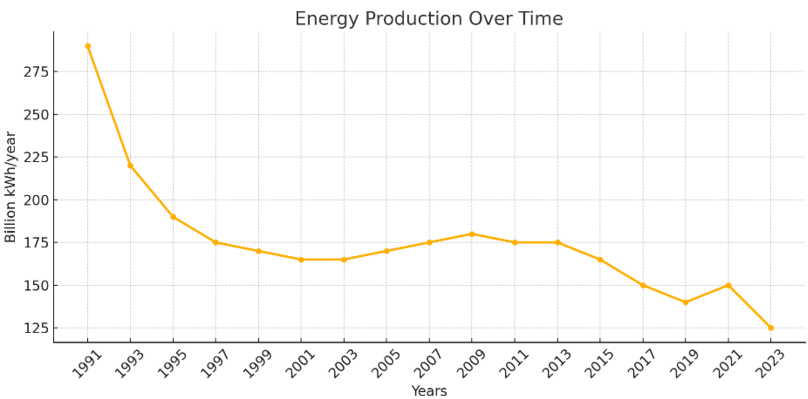
In fact, when redesigning the PSS structures, it was planned only to “increase” the number of and additional PL segments. At the same time, the real logic of the influence of these changes on the electric power parameters of the generalized PSS functioning in the required volume – sufficient for a preventive assessment of the expected changes, while EQI was not analyzed. Accordingly, for various reasons, there was no timely planning to prevent negative changes in the technology of the electric power industry functioning of these enterprises, which logically should have deteriorated, and did deteriorate as a result of such changes. That is, additional EQI were added to the negative levels of the existing EQI.

The general "today" state of the energy intensity that is adequate to the energy efficiency of mining enterprises is determined primarily by the technology of their operation, as well as outdated, from the point of view of modernity, PSS, and electrical equipment that has practically long exhausted its technical resources. It is impossible to influence the first of the above components from the energy side, but it is real, it is necessary and it is implemented to a certain extent on the others. However, the achieved successes are leveled by the processes associated with the above-mentioned process of constant deepening of IO extraction. That is, this direction, being necessary, has its technological limitations.

For known reasons, since 2014, both in the electric power structure: generation - distribution - consumption of electricity, and in the mining and metallurgical industry of the state, a number of additional negative aspects have arisen in the sphere of the efficiency of their individual and aggregate functioning [24].

Fig. 1.1 shows the production volumes, and Fig. 1.2 shows the installed electricity generation capacities in Ukraine [16-19]. It is obvious from the figure that the electricity production rate in the country has decreased by almost 40% in recent years. At the same time, electricity consumption by industrial enterprises of the country has decreased by approximately 35% (Fig. 1.3). In a certain sense, the comparison of the first and second indicators bears the sign of imbalance.

This is a fact of the unstable functioning of the Ukrainian electricity system in general and its components in particular, which is directly reflected in the work of mining enterprises, since February 2022, when, as a result of the shutdown of a number of electricity generation entities, the state and its basic enterprises in the national economy are limited by limits on daily consumption levels of this type of energy. In addition to these restrictions, enterprises are forced to take into account the variability of "rigid" hourly electricity tariffs, which are characterized by their constant hourly and daily variability (Fig. 1.7). Thus, at certain hours of the day, the hourly difference in payment (tariffs) for electricity by consumers reaches almost 20 times. An approximation to such a situation is observed in most cases for a significant period of time.



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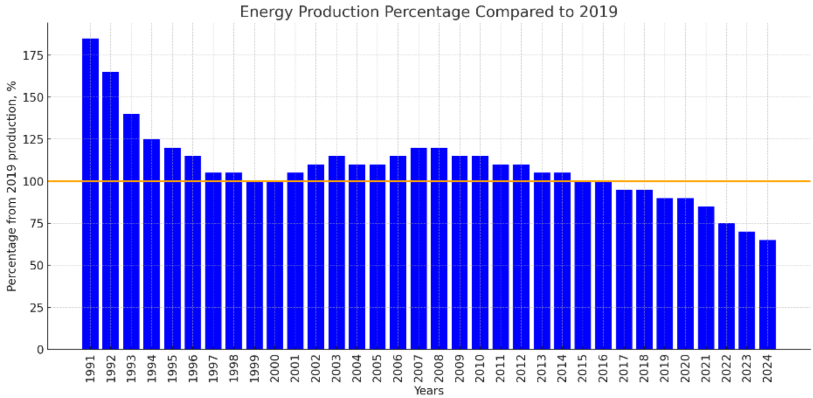

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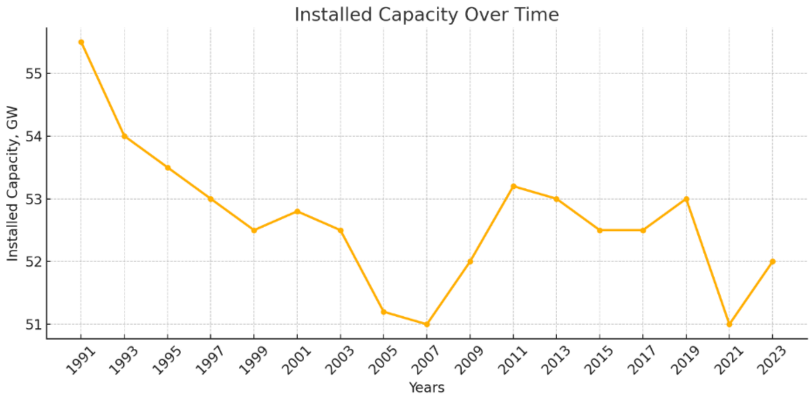
Figure 1.1 – Electricity generation volumes in Ukraine by year (a) and compared to 2019 (b)



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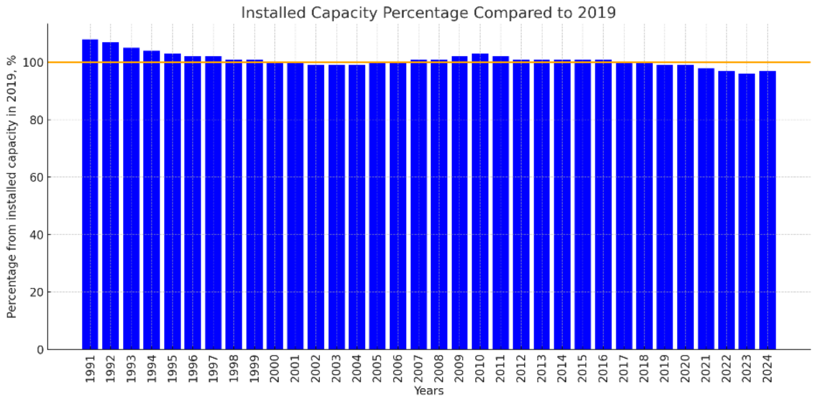

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Figure 1.2 – Installed capacity of power plants in Ukraine in different years (a) and compared to 2019

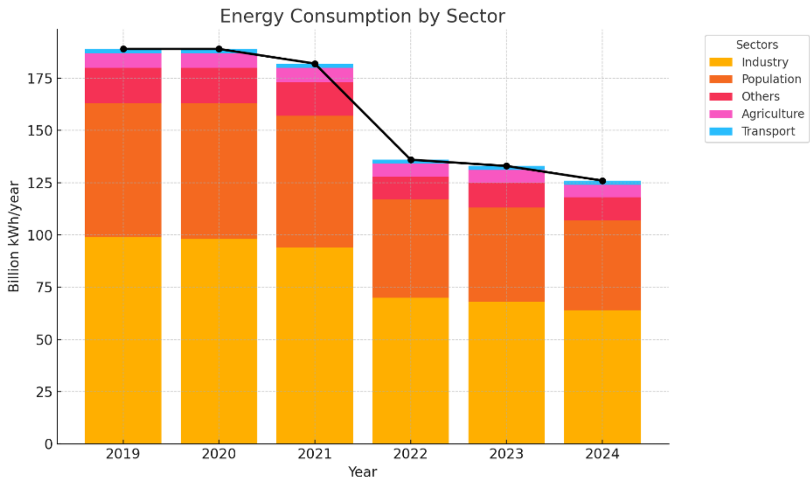


Figure 1.3 – Electricity consumption volumes by various sectors in Ukraine for 2019-2024

At the same time, the maximum values of tariff levels belong to the periods of the day between 18.00 - 23.00. The level of compliance of supply and demand for the analyzed day fluctuated in the range of "check and balance" from -2007.2 to +804.9 MWh., which until February 2022 was not a typical phenomenon, but was precisely the opposite [17].

This, along with a number of other problems, is significantly supplemented by the fact of a decrease in production volumes - IO production at the analyzed types of enterprises (Fig. 1.4), which was clearly reflected in the production of pig iron (Fig. 1.5) - the primary variant of metal distribution. Since 2023, the trend of increasing the cost of iron ore has also resumed (Fig. 1.6), although the instability and unpredictability of pricing inherent to this indicator continues here. The electricity component of price formation also makes its negative contribution to this process, which has acquired a constant status in its growth [16-19].

For greater clarity and specificity, we note that as of the beginning of 2022, electricity consumers in the Dnipropetrovsk region account for about 30% of the national consumption level, the majority of which goes to the needs of the region's mining and metallurgical industries, including the Kryvyi Rih iron ore basin [17, 19].

It is still trivial, but we note that there has been no stability in the functioning of mining enterprises in the state since 2022, and, moreover, a number of enterprises in the industry did not mine iron ore at all in some months of 2023-2024. At the same time, electricity consumption by these enterprises in such cases did not stop. The latter fact, in its drama, complements the complex of troubles in the power sector of mining enterprises and, accordingly, in their economy. This requires a creative approach in new modern formats for solving the problem of increasing the energy efficiency of these types of industrial enterprises.

However, returning to the "troubles" in the energy system of Ukraine, let's add one more - a systemic cohort of problems in this sphere of the state's life in general and the mining industry in particular - electricity losses.

Electricity losses during transportation along main PLs in Ukraine for the period of 2019 were determined at the level of 10.5 billion kWh. that is, about 7.5% of the total transported volume [19]. However, a number of scientific publications give loss figures that are at least twice as high. It is logical that as of the beginning of 2025, these indicators did not deviate towards the positive, but rather the opposite.

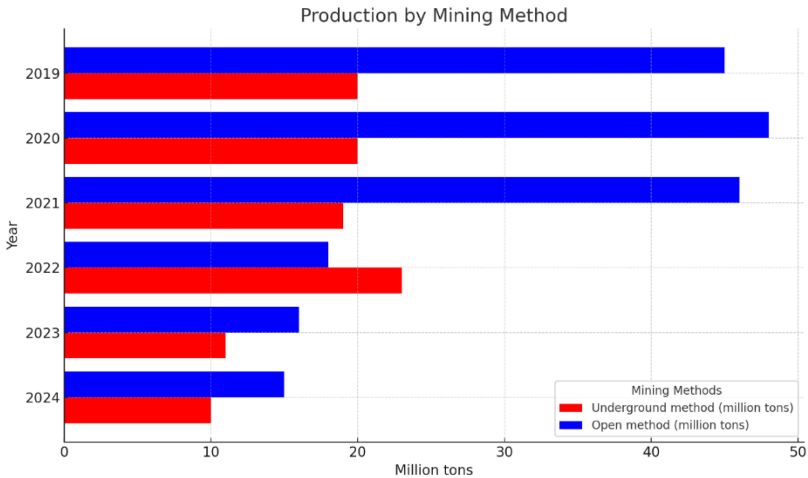


Figure 1.4 – Iron ore production volumes in Ukraine for 2019-2024

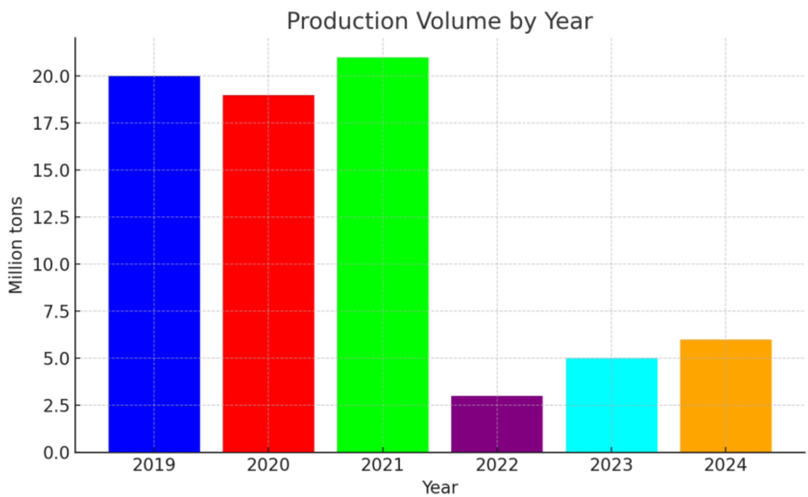


Figure 1.5 – Production of cast iron in Ukraine for 2019-2024

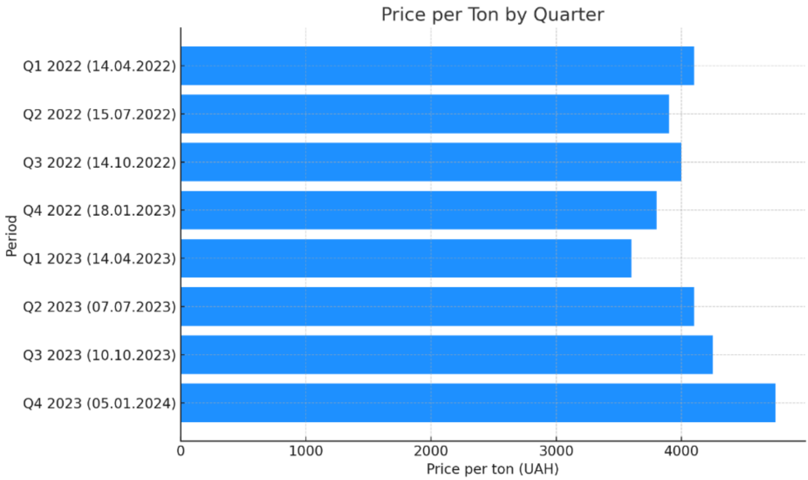


Figure 1.6 – Cost of iron ore mined for the quarterly periods 2022-2024

Figure 1.7 – Hourly electricity purchase and sale prices on the day-ahead market

As a supplement to this thesis, we add that according to the research of scientists of the Institute of Electrical Engineering of the National

Academy of Sciences of Ukraine, every 100 km. PL 220 kV generate 13 MVAr of power and, accordingly, for PL 330 and 500 kV these values are 39 MVAr and 96 MVAr, respectively. And at half load PL 750 kV with a length of 400 km it generates about 700 MVAr of reactive power into the network, and at idle - 900 MVAr, which must be compensated [25-27].

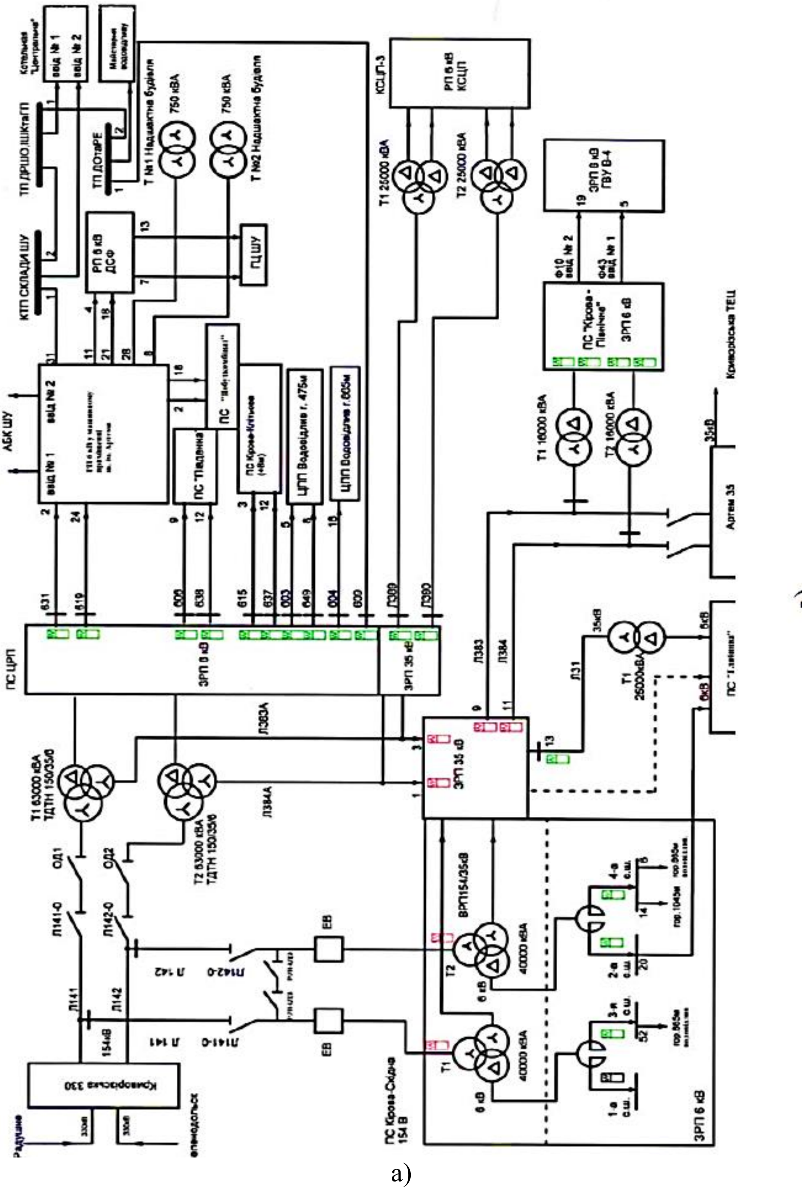
From the above, a logical conclusion is formed that to ensure the normal functioning of the electrical system of Ukraine, it is necessary to maintain a balance between the levels of generation and consumption of electricity.

1.2. Power supply systems and modes of electricity consumption by receivers of underground mines. Condition, problems

As was noted in the previous section of this study, the method, or rather the technology of mineral extraction, determines the format of PSS development of these types of enterprises. As for the intra-industrial structures of PSS of underground mining enterprises (mines, shafts), structurally they, as a rule, exist without significant changes from the moment the enterprise is put into operation until its closure, and often after this process, if the enterprise is in the stage of its conservation. The only types of changes, as for intra-mine PSS, are those that concern the process of their "building up" - additions, which is associated with the process of reducing the levels of IO extraction and other insignificant additions in the possible change in the technological capabilities of the functioning of a particular enterprise.

As a rule, intra-mine PSS are developed in the variant of radial and mixed structures [26]. The scheme of power supply of an operating underground mine (as a mine) from IO extraction is shown in Fig. 1.8.

A characteristic and system-forming feature of the functioning of the PSS-based types of mining enterprises is that the operating modes of their power complexes are continuous throughout the day, but the levels of fluctuations in the volume of electricity consumption are significant compared to the average daily. This nature of fluctuations is both natural - according to the technology of operation of these enterprises, and with a certain bias, the result of manual control [27-29].



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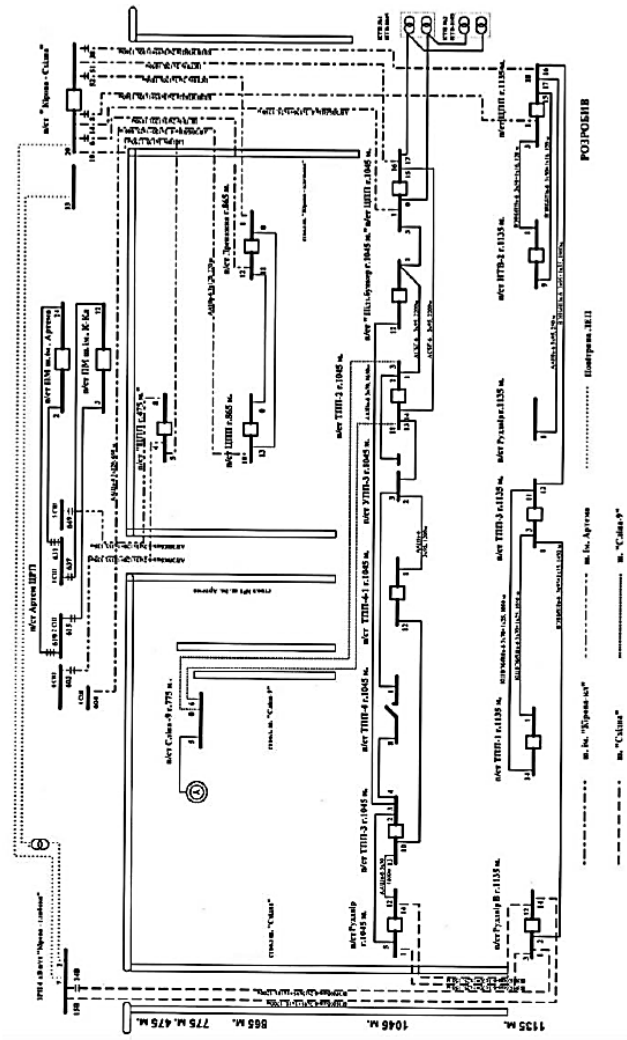


b)

Figure 1.8 – Power supply schemes of the underground mine (as mine No. 9): a) external power supply scheme; b) scheme of underground 6 kV cable networks.

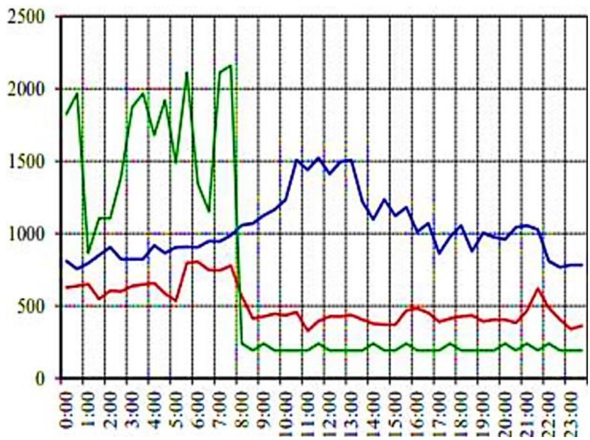
Moreover, the formats of daily fluctuations even for the same mine do not repeat from day to day. In Fig. 1.9, for the example of an operating underground mine, daily electricity consumption graphs are shown.

As a supplement to the analysis of daily electricity consumption graphs by underground mines producing IO, we add that a significant part of the total electricity consumption of these enterprises belongs to receivers that are not direct participants in the IO extraction process, and the level of their electricity consumption does not depend on the current production volumes [30].

Fig. 1.10 provides an example of this statement illustrating the indicators of iron ore production and the corresponding energy consumption by a specific operating mine - on the basis of the mine rights - of the Arcelor Mittal Kryvyi Rih division.

Apparently, this is yet another confirmation of the well-known fact that electricity consumption does not directly correlate with the volume of IO production and the process of electricity consumption by them continues even when the mining enterprise is not working on ore extraction.

For the economy of the enterprise, this is a drama, which is amplified by the fact that the quality of electricity indicators deteriorates during the operation of the enterprise [31, 32]. That is, the enterprise, paying for the volume of consumed electricity, spends a significant part of these material costs on electricity that it does not actually use and which was lost during its transportation.



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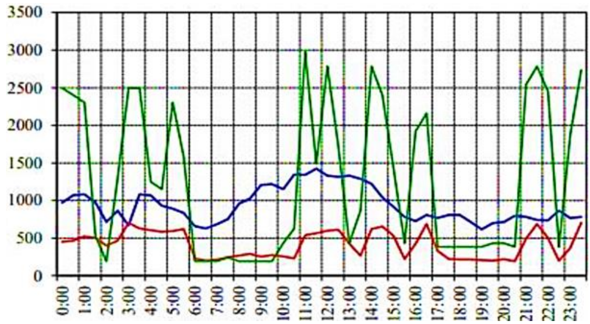

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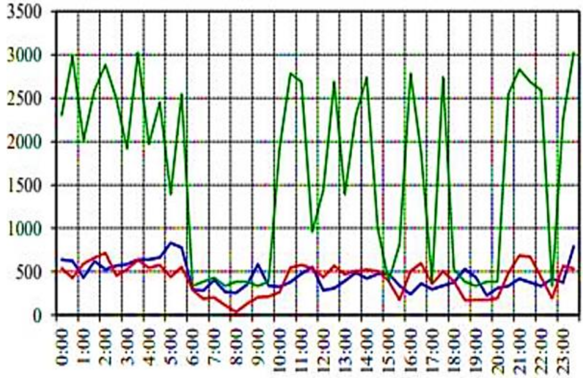

c)

Figure 1.9 – Electricity consumption by individual receivers of the underground iron ore mine under the rights of mine No. 8, Kryvyi Rih (-------- – in general; -------- – crushing and sorting plant; ------- – skip hoisting plant):
a) as of 01/31/2022; b) as of 03/01/2022; c) as of 04/30/2022

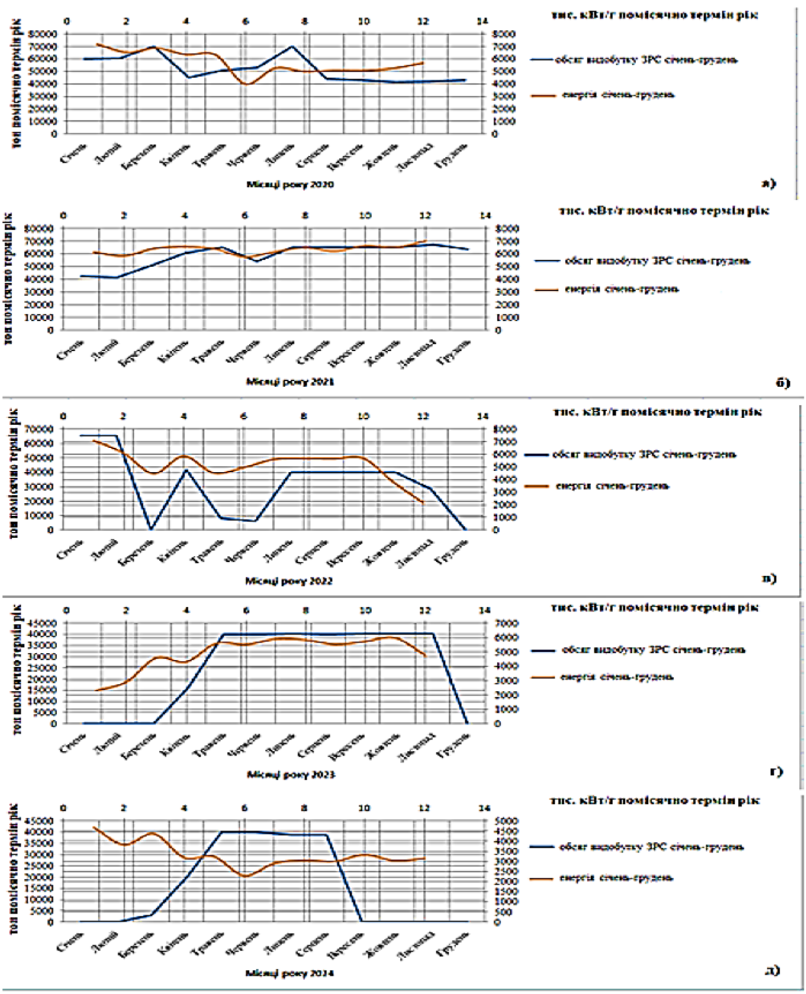

Figure 1.10 – Monthly electricity consumption levels and iron ore production volumes by an underground mine under the rights of a mine for 2020-2024 (mine No. 8, Kryvyi Rih): a) 2020; b) 2021; c) 2022; d) 2023; e) 2024

However, and what is one of the other, in addition to the losses of electricity in the complex of system-forming levers, the need to comply with standardized values, is that the deviation of the actual indicators (parameters) of the functioning of the PSS from the nominal values, significantly affects the service life of their components: power transformers, electric motors and other electrical equipment [Circle]. In the calculated version, such compliance is ensured by compliance with the three-unit condition:

$$S_{\text{мер}} = S_{\text{н}};\ U_{\text{мер}} = U_{\text{н}};\ \tau_{\text{мер}} = \tau_{\text{н}}$$

where $S_{\text{мер}}$; $S_{\text{н}}$ - respectively, the actual operating load and the nominal (passport) electrical power; $U_{\text{мер}}, U_{\text{н}}$ - corresponding mains voltage and nominal (passport) voltage of the receiver; $\tau_{\text{мер}}, \tau_{\text{н}}$ - respectively, the ambient temperature where the receiver is installed and operating, and the limit of its permissible values according to the receiver's passport.

If we add to this, as a supplement, the fact that the deviation of EQI from the warranty conditions standardized by the manufacturers of this or that electrical equipment significantly affects their functional and energy indicators, then the unsightly and ultimately unpredictable situation in the electric power industry of enterprises - consumers of electricity is highlighted. This fully applies to the power transformers of the corresponding PSS substations of these enterprises.

One of the modern systemic reasons for low EQI in the PSS of mining enterprises is the underload of the transformers of the of the analyzed types of enterprises.

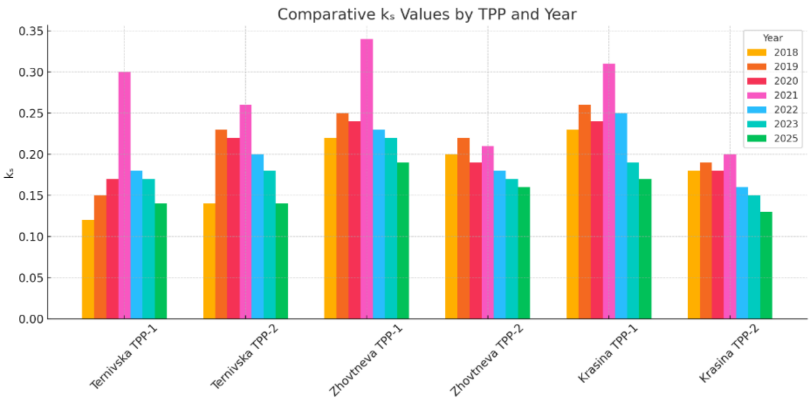

Figure 1.11 – Transformer load levels of a number of main step-down substations of underground mines of the Kryvyi Rih Iron Ore Basin (2024)

Thus, analyzing in a preventive, but sufficient for assessing the current state of the electric power industry of underground mines with IO extraction, the statement that the state is practically dramatic seems certain. However, despite a number of system-forming factors of influence that do not relate to the electric power industry, but which determine the level of its functioning, there are still real opportunities to improve the electric power indicators of the electric power systems of these types of enterprises. One of such directions is to solve the problem of maintaining the current EQI levels at standard levels, which will make it possible to reduce electricity losses in the internal PSS of mines and thereby improve the quality of its use and reduce material costs for the purchase of this type of energy.

SECTION 2.
STUDY OF ELECTRICITY QUALITY IN THE PARADIGM OF MANAGERIAL ACTIONS[1]

2.1 Simulation of power supply quality under conditions of electricity consumption by receivers of underground mines

The quality of electric energy - a standardized EQI complex, as well as the quality of electricity supply - a non-standardized definition, are interconnected and mutually influencing related concepts that form input parameters for the effective operation of both specific electricity consumers and the corresponding electric power complexes as a whole.

Going within the logistics of developing a search structure variant: determining the state - determining and assessing influence factors - developing an EQI control system within the permissible limits of standard levels, it is advisable to develop an appropriate research model, which, based on real input parameters, which are based on the corresponding experimental measurements of operating productions, will make it possible to rebuild the logic of the control algorithm for this process with the subsequent development of the corresponding management system.

Based on the established fact [12-14] that fluctuations in electricity consumption levels in underground iron ore mines are stochastic in time, which is accordingly reflected in the EQI parameters, including the levels of voltage deviations in the PSS and receiver terminals, for further search, in accordance with the structurally logical format of achieving the goal - controllability of this process, a solution involving one of the effective processes for data analysis, against the background of existing options - the apparatus of mathematical modeling based on "emission problems" [35] seems reasonable.

2.1.1 Input comments on the disclosure of research aspects

Consider the process of electricity consumption $W(t)$ as a random function ε, i.e. a function of its argument whose value at any time *t* is a

[1] Professor Beridze Tatyana Mikhailovna participated in the formation of the section's structure in an advisory format.

random variable. If the argument of a random function *t* takes on any values in a given interval, then the random function will be called a random process.

The process $W(t)$ is called continuous in probability if for any ε ($\varepsilon > 0$) there is

$$\lim_{\Delta \to 0} P\left\{\left|W(t+\Delta) - W(t)\right| \geq \varepsilon\right\} = 0.$$

Consider a random function $W(t)$ and assume that n independent experiments are conducted to study it. Each experiment will yield a realization of the random function $W(t)$. A random function can be considered given if all multivariate distribution laws of $f(x_1, x_2, ..., x_n | t_1, t_2 ..., t_n)$ are given for any values of $t_1, t_2 ..., t_n$ in the domain of variation of the argument t.

However, the construction of a multidimensional distribution law is inconvenient, since it is cumbersome. Therefore, in practice, instead of the multidimensional distribution laws themselves, they are limited to specifying the corresponding numerical parameters of these laws. The most convenient such parameters are the initial or central moments of different orders. Of the infinite number of moments, the most important from the point of view of the characteristics of a random function are the moments of the first and second order. The moment of the first order, which is determined by the formula [36, 37]

$$m_1 = M\left[W(t_1)\right], \tag{2.1}$$

is the mathematical expectation of the ordinate of a random function at an arbitrary time. The mathematical expectation (2.1) depends on the time value and shows the average value of A, therefore it is denoted as

$$\overline{w}(t) = M\left[W(t)\right]. \tag{2.2}$$

Function (2.2) is not random and is completely determined by the one-dimensional probability density

$$\overline{w}(t) = \int_{-\infty}^{\infty} wf(w|t)dw. \tag{2.3}$$

The initial second-order moments can be of two types: the second-order moments of one of the ordinates of a random function, defined by the formula

$$m_2 = M\left[W^2(t_1)\right], \tag{2.4}$$

and mixed second-order moments

$$m_{1,1} = M\left[W(t_1)W(t_2)\right]. \tag{2.5}$$

Instead of initial moments, second-order central moments are more often used, which are determined by the equalities

$$D\left[W(t)\right] = M\left\{\left[W(t) - \bar{w}(t)\right]^2\right\}, \tag{2.6}$$

$$K(t_1, t_2) = M\left\{\left[W(t_1) - \bar{w}(t_1)\right]\left[W(t_2) - \bar{w}(t_2)\right]\right\}. \tag{2.7}$$

According to (2.6) and (2.7), $D\left[W(t)\right]$ is the variance of the random variable , and $K(t_1, t_2)$ is the correlation function.

Explicit expressions for the mathematical expectation for the variance (2.6) and the correlation function (2.7) are written in the form

$$D\left[W(t)\right] = \int_{-\infty}^{\infty}\left[w - \bar{w}(t)\right]^2 f(w|t)dw, \tag{2.8}$$

$$K(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[w_1 - \bar{w}(t_1)\right]\left[w_2 - \bar{w}(t_2)\right] f(w_1, w_2|t_1, t_2)d\,w_1 dw_2. \tag{2.9}$$

The section of the theory of random functions that operates only with the moments of the first two orders is called the correlation theory of random functions [39].

When solving the problem of emissions, the correlation theory of random functions will be used.

The most important property of a random function, which has found application in solving the problem of emissions, is the independence of the properties of the random function from the start of time. Accordingly, the concept of stationary random functions is introduced. For stationary random functions, all multivariate distribution laws depend not only on the relative location of the moments of time, but on the values of these quantities themselves. That is, the relationship must hold

$$f(w_1, w_2, ..., w_n | t_1, t_2, ..., t_n) = f(w_1, w_2, ..., w_n | t_1 + t_0, t_2 + t_0, ..., t_n + t_0), \quad (2.10)$$

where t_0 – any number.

In a special case, for $n=1$ and $n=2$, assuming $t_0 = -t_1$ according to (2.10), we will have

$$f(w_1 | t_1) = f(w_1 | 0) = f(w_1), \quad (2.11)$$

$$f(w_1, w_2 | t_1, t_2) = f(w_1, w_2 | 0, t_2 - t_1) = f(w_1, w_2 | t_2 - t_1). \quad (2.12)$$

Substituting (2.11) and (2.12) into (2.3), (2.8) and (2.9), we get

$$\overline{w}(t) = \int_{-\infty}^{\infty} w f(w) dw = \overline{w}, \quad (2.13)$$

$$D[W(t)] = \int_{-\infty}^{\infty} (w - \overline{w})^2 f(w) dw = D(W), \quad (2.14)$$

$$K(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [w_1 - \overline{w}][w_2 - \overline{w}] f(w_1, w_2 | t_2 - t_1) d w_1 dw_2 = K(t_2 - t_1). \quad (2.15)$$

If conditions (2.13), (2.14), and (2.15) are satisfied, then the random function $W(t)$ will be called stationary in a broad sense.

Another feature that can be used to classify random functions is the type of distribution laws of the ordinates of a random function. The most common distribution law is the normal distribution law (Gauss's law) [39]. In the case of a single random variable, the normal distribution law takes the form

$$f(w) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(w-\overline{w})^2}{2\sigma^2}}, \quad (2.16)$$

where $\sigma^2 = D(W)$.

For the integral distribution function according to (2.16), the formula holds

$$F(w)=\int_{-\infty}^{w} f(w)dw=\frac{1}{2}\left[1+\Phi\left(\frac{w-\overline{w}}{\sigma}\right)\right], \tag{2.17}$$

where $\Phi\left(x\right)=\sqrt{\frac{2}{\pi}}\int_{0}^{x} e^{-\frac{t^2}{2}}dt-$ Laplace integral function.

The study of normal random functions is facilitated by the fact that to determine the distribution law of a system of normal quantities, it is sufficient to know only their mathematical expectations and correlation functions.

A random process $W(t)$ is differentiable if there exists a limit

$$\lim_{\varepsilon\to 0}\frac{W(t+\varepsilon)-W(t)}{\varepsilon}=\frac{dW(t)}{dt}, \tag{2.18}$$

which is called the derivative of the process $W(t)$ at point t.

In order for a random function to have a derivative, it is sufficient for the second derivative of the correlation function to exist for equal values of its arguments. For stationary random functions, the following holds:

$$K(t_1,t_2)=K(t_2-t_1)=K(\tau), \tag{2.19}$$

therefore, according to (2.19),

$$\frac{\partial^2 K(t_2-t_1)}{\partial t_1 \partial t_2}=\Big|_{t_1=t_2=t}=-\frac{d^2K(\tau)}{d\tau^2}\Big|_{\tau=0}. \tag{2.20}$$

Note that the correlation function of the derivative of a random function is equal to the second mixed partial derivative of the correlation function of the differentiable random function, i.e.

$$K_v(t_1,t_2)=\frac{\partial^2 K_w(t_1,t_2)}{\partial t_1 \partial t_2}. \tag{2.21}$$

In the case when the random function $W(t)$ is stationary, instead of (2.21) we have

$$K_v(\tau)=-\frac{d^2K_w(\tau)}{d\tau^2}. \tag{2.22}$$

After a brief introduction to random functions and their properties, we will consider solving the problem of emissions when studying the quality of electricity supply at mining enterprises.

Let $W(t)$ be the power consumption of electricity, which is a differentiable stationary random process. Let β be the value of the power consumption of electricity $W(t)$ for which emissions are investigated.

We will call the intersection of a process $W(t)$ of a given level with a positive derivative (from bottom to top) a positive outlier, from top to bottom a negative outlier. Confining ourselves to positive outliers, we set the task of determining their following probabilistic characteristics:

• numerical characteristics of the random number of emissions over the time interval T;

• numerical characteristics of a random time in the interval T during which $W(t)$ exceeds a given level β.

Let us find the mathematical expectation of the number of positive emissions over the time interval T. Let us determine the probability that an emission will occur in an infinitely small time interval dt following the time instant t. In order for an emission to occur under the specified conditions, two events must occur: first, at time instant t the ordinate of the random function must be less than β, i.e.

$$W(t) < \beta, \tag{2.23}$$

and, secondly, at time $t + dt$ the ordinate of the random function must be greater than β, i.e.

$$W(t+dt) > \beta. \tag{2.24}$$

Therefore, the probability of emission in time interval dt can be written as

$$P\left[W(t) < \beta; W(t+dt) > \beta\right]. \tag{2.25}$$

Using the condition of differentiability of the random function $W(t)$, the inequality (2.25), which imposes restrictions on the ordinates of the random function $W(t)$ at two adjacent points, can be replaced by inequalities imposed on the ordinate of the random function $W(t)$ and its derivative $V(t)$ at one point. Indeed, taking into account the smallness of the time interval dt with an accuracy of infinitesimals of the second order, we obtain

$$W(t+dt)=W(t)+V(t)dt\,,\quad V(t)=\frac{dW(t)}{dt}\,. \tag{2.26}$$

So, the inequality

$$W(t+dt)>\beta \tag{2.27}$$

equivalent to inequality

$$W(t)+V(t)dt>\beta\,,$$

or

$$\beta-V(t)dt<W(t)\,. \tag{2.28}$$

Instead of the two inequalities (2.23) and (2.24), which determine in (2.25) the probability of the presence of an emission in the time interval dt, one can write one double inequality

$$\beta-V(t)dt<W(t)<\beta\,,\quad \left(V(t)>0\right)\,. \tag{2.29}$$

To calculate the probability of inequality (2.29), it is necessary to introduce a two-dimensional distribution law of the ordinate of the random function $W(t)$ and its derivative $V(t)$ at the same time point t

$$f(w,v|t)\,. \tag{2.30}$$

Then for the probability of emission we get

$$P\left[\beta-V(t)dt<W(t)<\beta\right]=\int_0^{\infty}\int_{\beta-V(t)dt}^{\beta}f(w,v|t)dwdv\,, \tag{2.31}$$

where the limits of integration cover all values of $W(t)$ and $V(t)$ satisfying inequalities (2.29). The inner integral (2.31) can be calculated, since its limits of integration differ by an infinitesimal value $v\cdot dt$, and using the mean value theorem, we obtain

$$\int_{\beta - vdt}^{\beta} f(w, v|t)dw = dt \cdot v \cdot f(\beta, v|t) \quad . \qquad (2.32)$$

Substituting (2.32) into (2.31) gives

$$P[\beta - V(t)dt < W(t) < \beta] = dt\int_{0}^{\infty} f(\beta, v|t)vdv \quad . \qquad (2.33)$$

Formula (2.33) shows that the possibility of an emission during an infinitely small time interval dt is proportional to the size of this interval. Therefore, it is advisable to introduce the concept of time density for the probability of emission, denoting $p(\beta|t)$ the probability of emission for level β at time t, calculated per unit of time, i.e., putting

$$P[\beta - V(t)dt < W(t) < \beta] = p(\beta|t)\,dt \quad . \qquad (2.34)$$

Comparing (2.34) with (2.33) gives the final expression for the probability density $p(\beta|t)$

$$p(\beta|t) = \int_{0}^{\infty} f(\beta, v|t)vdv \quad . \qquad (2.35)$$

Similarly, the time probability density $p'(\beta|t)$ of the intersection by a random function of level β from top to bottom can be calculated. Repeating the above considerations, we obtain

$$p'(\beta|t) = -\int_{-\infty}^{0} f(\beta, v|t)vdv \quad . \qquad (2.36)$$

Adding and subtracting (2.35) and (2.36), we obtain

$$p(\beta|t) + p'(\beta|t) = \int_{-\infty}^{\infty} f(\beta, v|t)|v|\,dv \quad , \qquad (2.37)$$

$$p(\beta|t) - p'(\beta|t) = \int_{-\infty}^{\infty} f(\beta, v|t)vdv \quad . \qquad (2.38)$$

Using (2.35), we can obtain for the time interval T the average time the random function stays above a given level. Indeed, let us divide the time interval T into n equal-sized small intervals dt_j, each of which is located near the time instant t_j ($j = 1, 2, ..., n$). The probability that the ordinates of a random function $W(t_j)$ will be higher than a given level β is determined by the formula

$$P(W(t_j) > \beta) = \int_{\beta}^{\infty} f(w|t_j)dw \quad . \tag{2.39}$$

We assume that the values of the intervals dt_j are so small that when calculating the total time a random function $W(t_j)$ stays above a given level β, we can neglect cases where the function $\left[W(t_j) - \beta\right]$ changes sign within the interval. Let us consider a system of random variables Δ_j, each of which is equal to the corresponding interval dt_j or 0, depending on whether the random function $W(t_j)$ is greater or less than β in this interval. Then the total time T_β of the random function $W(t)$ above the given level β is equal to the sum Δ_j, i.e.

$$T_\beta = \sum_{j=1}^{n} \Delta_j \quad . \tag{2.40}$$

To determine the average time $\overline{t_\beta}$ of a random function $W(t)$ above a given level β during time T, it is necessary to find the mathematical expectation of both sides of equality (2.40). Applying the theorem on the mathematical expectation of the sum, we find

$$\overline{t_\beta} = \sum_{j=1}^{n} M\left[\Delta_j\right] . \qquad (2.41)$$

The random variable Δ_j takes only two values (dt_j and 0), therefore its mathematical expectation is equal to

$$M\left[\Delta_j\right] = dt_j \int_{\beta}^{\infty} f(w|t_j)dw . \qquad (2.42)$$

Substituting (2.42) into (2.41) and going to the limit at $n \to \infty$ instead of the sum, we obtain an integral, and for the average time of the random function $W(t)$ above the level β, calculated for the time interval T, we obtain

$$\overline{t_\beta} = \int_0^T \int_{\beta}^{\infty} f(w|t)dwdt . \qquad (2.43)$$

In practice, the average time a random function spends above a given level during a single outburst is of interest. To determine this average time $\overline{\tau}$ (2.43) must be divided by the average number of outbursts $\overline{n}_\beta$ that occurred during time T. To determine the average number of emissions $\overline{n}_\beta$, it is necessary to divide the interval T into n equal intervals dt_j and introduce auxiliary values N_j, each of which is equal to one if an emission occurred within the corresponding interval (due to the small length of the intervals dt_j, the possibility of more than one emission can be disregarded), and zero in the opposite case. Then the total number of emissions N_β over a time period T will be equal to the sum of the quantities N_j

$$N_\beta = \sum_{j=1}^{n} N_j . \qquad (2.44)$$

Finding the mathematical expectation of both parts of equality (2.44) and taking into account that the mathematical expectation of each of the quantities N_j is numerically equal to the probability of an outlier in the j-th interval

$$M\left[N_j\right] = p(\beta|t_j)dt_j$$

that is, there will be a

$$\overline{n}_\beta = \sum_{j=1}^{n} p(\beta|t_j)dt_j \quad . \tag{2.45}$$

Increasing the number of intervals dt_j to infinity and replacing the sum with the integral and substituting (2.35), we obtain

$$\overline{n}_\beta = \int_0^T \int_\beta^\infty v f(\beta, v|t) dv dt \quad . \tag{2.46}$$

Dividing (2.43) by (2.46) gives the desired average duration of the emission

$$\overline{\tau}_\beta = \frac{\overline{t}_\beta}{\overline{n}_\beta} = \frac{\int_0^T \int_\beta^\infty f(w|t) dw dt}{\int_0^T \int_\beta^\infty v f(\beta, v|t) dv dt} \quad . \tag{2.47}$$

The obtained formulas are of greatest interest for stationary processes, because for processes that have stabilized in time, the average duration of the emission is of obvious importance. For stationary processes, these formulas are simplified because the density of the ordinate distribution of the random function $f(w|t)$ and the density of the ordinate and velocity distribution of $f(w,v|t)$ do not depend on time. Denoting these distribution densities by $f(w)$ and $f(w,v)$, we note that integration over time is reduced to multiplication by the time interval T and for the average time of the stationary random function $W(t)$ above the given level β during the time T (2.43), the average number of emissions over the same time interval T (2.46) and the average duration of the emission (2.47) we obtain the formulas

$$\overline{t_\beta} = T\int_\beta^\infty f(w)dw, \qquad (2.48)$$

$$\overline{n}_\beta = T\int_\beta^\infty vf(\beta,v)dvdt, \qquad (2.49)$$

$$\overline{\tau}_\beta = \frac{\int_\beta^\infty f(w)dw}{\int_\beta^\infty vf(\beta,v)dv}. \qquad (2.50)$$

For stationary processes, the concept of the average number of emissions per unit of time can be introduced [39]

$$\overline{v}_\beta = \frac{\overline{n}_\beta}{T}. \qquad (2.51)$$

Taking into account (2.49), formula (2.51) takes the form

$$\overline{v}_\beta = \int_\beta^\infty vf(\beta,v)dvdt. \qquad (2.52)$$

Since all the above formulas include probability densities, it is necessary to have these densities to obtain numerical results.

For the normal process, which is the most important from a practical point of view, fairly simple calculation formulas can be obtained.

For a normal stationary process, the distribution law of the ordinates of a random function is uniquely expressed in terms of the mathematical expectation of the random function $\overline{w}$ and its variance

$$\sigma_w^2 = K_w(0), \qquad (2.53)$$

because

$$f(w) = \frac{1}{\sigma_w\sqrt{2\pi}} e^{-\frac{(w-\overline{w})^2}{2\sigma_w^2}}. \qquad (2.54)$$

The rate of change of the ordinate of a random function and the ordinate of the random function for the same moment in time are uncorrelated random variables, and for a normal random process, they are also independent variables. Therefore, the two-dimensional probability density distribution $f(w,v)$ decomposes into the product of the normal distribution densities for W and V, we can write

$$f(w,v)=\frac{1}{\sigma_w\sqrt{2\pi}}e^{-\frac{(w-\overline{w})^2}{2\sigma_w^2}}\cdot\frac{1}{\sigma_v\sqrt{2\pi}}e^{-\frac{v^2}{2\sigma_v^2}}. \qquad (2.55)$$

The variance of the rate of change of the ordinate of a random function is equal to

$$\sigma_v^2=-\frac{d^2}{d\tau^2}K_w(\tau)\Big|_{\tau=0}. \qquad (2.56)$$

The mathematical expectation of the rate of change of the ordinate of a random function due to the stationarity of the random process is zero, i.e. $\overline{v}=0$.

Substituting (2.55) into (2.52) gives for the average number of emissions per unit time $\overline{v}_\beta$, or for the temporal probability density $p(\beta|t)=p(\beta)$

$$\overline{v}_\beta=p(\beta)=\frac{\sigma_v}{2\pi\sigma_w}e^{-\frac{(\beta-\overline{w})^2}{2\sigma_w^2}}. \qquad (2.57)$$

Similarly, after substitution in (2.50) we will have

$$\overline{\tau}_\beta=\pi\frac{\sigma_w}{\sigma_v}e^{-\frac{(\beta-\overline{w})^2}{2\sigma_w^2}}\left[1-\Phi\left(\frac{\beta-\overline{w}}{\sigma_w}\right)\right], \qquad (2.58)$$

where $\Phi(w)-$ Laplace integral function.

We will show how to find the average electrical energy $\overline{e}$, limited by the realization of a random function above a given level β during the emission, assuming the random function $W(t)$ to be stationary. For this purpose, we derive an auxiliary function, which is defined by the equality

$$I(w)=\begin{cases}1, & w>0\\ \frac{1}{2}, & w=0\\ 0, & w<0\end{cases}. \qquad (2.59)$$

Then the electrical energy of all emissions E that occurred during time T can be represented as

$$E=\int_0^T I\left[W(t)-\beta\right]\cdot W(t)dt-\beta\cdot\overline{\tau}_\beta\cdot\overline{v}_\beta\cdot T. \qquad (2.60)$$

The first factor in the integrand (2.60) is equal to one during the ejection and equal to zero in all other cases. Let us express the function (2.59) explicitly. For this we use the Dirichlet integral

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin uw}{u}du=\begin{cases}1, & w>0\\ \frac{1}{2}, & w=0\\ 0, & w<0\end{cases}.$$

If we consider the integral

$$\frac{1}{\pi\cdot i}\int_{-\infty}^{\infty}e^{i\cdot u\cdot w}\frac{du}{u}$$

in the sense of its main meaning, then

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin uw}{u}du=\frac{1}{\pi\cdot i}\int_{-\infty}^{\infty}e^{i\cdot u\cdot w}\frac{du}{u}. \qquad (2.61)$$

Indeed, since in (2.61) the integral is understood in the sense of the principal value, then

$$\frac{1}{\pi\cdot i}\int_{-\infty}^{\infty}e^{i\cdot u\cdot w}\frac{du}{u}=\frac{1}{\pi\cdot i}\lim_{\substack{\varepsilon\to0\\ L\to\infty}}\left\{\int_{-L}^{-\varepsilon}e^{i\cdot u\cdot w}\frac{du}{u}+\int_{\varepsilon}^{L}e^{i\cdot u\cdot w}\frac{du}{u}\right\},$$

where ε and L – positive quantities that tend to their limits independently of each other. Let us represent

$$e^{i\cdot u\cdot w} = \cos uw + i\cdot\sin uw.$$

Because

$$\lim_{\substack{\varepsilon\to 0\\ L\to\infty}}\left\{\int_{-L}^{-\varepsilon}\cos uw\frac{du}{u}+\int_{\varepsilon}^{L}\cos uw\frac{du}{u}\right\}=0,$$

then

$$\frac{1}{\pi\cdot i}\int_{-\infty}^{\infty}e^{i\cdot u\cdot w}\frac{du}{u}=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin uw}{u}du,$$

that is, formula (2.61) holds.

Taking into account (2.61), the function $I(w)$ can be represented in the form

$$I(w)=\frac{1}{2}\left[1+\frac{1}{\pi\cdot i}\int_{-\infty}^{\infty}e^{i\cdot u\cdot w}\frac{du}{u}\right]. \qquad (2.62)$$

Substituting (2.62) into (2.60), we obtain

$$E=\frac{1}{2}\int_{0}^{T}W(t)dt+\frac{1}{2\pi i}\int_{0}^{T}\int_{-\infty}^{\infty}e^{iu[W(t)-\beta]}W(t)\frac{du}{u}dt-\beta\cdot\overline{\tau}_{\beta}\cdot\overline{v}_{\beta}\cdot T. \qquad (2.63)$$

Let us apply the mathematical expectation operation to both parts of (2.63)

$$M[E]=M\left[\frac{1}{2}\int_{0}^{T}W(t)dt+\frac{1}{2\pi i}\int_{0}^{T}\int_{-\infty}^{\infty}e^{iu[W(t)-\beta]}W(t)\frac{du}{u}dt-\beta\cdot\overline{\tau}_{\beta}\cdot\overline{v}_{\beta}\cdot T\right] \qquad (2.64)$$

As a result, we will have, for the left-hand side of equality (2.64)

$$M[E]=\overline{s}\cdot\overline{v}_{\beta}\cdot T-\beta\cdot\overline{\tau}_{\beta}\cdot\overline{v}_{\beta}\cdot T.$$

For the right-hand side of equality (2.64)

$$M\left[\frac{1}{2}\int_{0}^{T}W(t)dt+\frac{1}{2\pi i}\int_{0}^{T}\int_{-\infty}^{\infty}e^{iu[W(t)-\beta]}W(t)\frac{du}{u}dt-\beta\cdot\overline{\tau}_{\beta}\cdot\overline{v}_{\beta}\cdot T\right]=$$

$$=\frac{1}{2}\int_{0}^{T}\overline{w}dt+\frac{1}{2\pi i}\int_{0}^{T}\int_{-\infty}^{\infty}e^{-i\beta u}M\left[e^{iuW(t)}W(t)\right]\frac{du}{u}dt-\beta\cdot\overline{\tau}_{\beta}\cdot\overline{v}_{\beta}\cdot T. \qquad (2.65)$$

The mathematical expectation, which stands in (2.65) under the integral sign, can be expressed in terms of the characteristic function $\overset{\smile}{E}(u)$ of the process $W(t)$, which is defined as

$$\overset{\smile}{E}(u) = M\left[e^{iuW}\right] = \int_{-\infty}^{\infty} e^{iuw} f_w(w)dw \quad , \tag{2.66}$$

where $f_w(w)$ – probability density of random variable W.

The convenience of using the characteristic function (2.66) is due to the fact that all moments of the random function can be obtained from the characteristic function (2.66) by differentiation. A number of calculations are easier to perform using not the probability density, but the characteristic functions, knowing which you can always find the probability densities of random variables by applying the inverse Fourier transform

$$f_w(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-iuw}\overset{\smile}{E}(u)du \quad . \tag{2.67}$$

Because

$$\frac{1}{i}\frac{d}{du}\overset{\smile}{E}(u) = \frac{1}{i}\frac{d}{du}\int_{-\infty}^{\infty} e^{iuw} f_w(w)dw = \int_{-\infty}^{\infty} e^{iuw} w\cdot f_w(w)dw = M\left[e^{iuW(t)}W(t)\right] \quad . \tag{2.68}$$

Substituting (2.68) into (2.65), integrating over t, since the integrands do not depend on time, and reducing both sides of the equality by time T, we obtain

$$\overline{s}\cdot\overline{v}_\beta = \frac{1}{2}\overline{w} - \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-iu\beta}\frac{d\overset{\smile}{E}(u)}{du}\frac{du}{u} - \beta\cdot\overline{\tau}_\beta\cdot\overline{v}_\beta \quad . \tag{2.69}$$

Integral

$$J = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\cdot u\cdot\beta}\frac{d\overset{\smile}{E}(u)}{du}\frac{du}{u} \tag{2.70}$$

can be expressed in terms of the probability density of $f_w(w)$. Indeed, considering the integral (2.70) as a function of β and differentiating both sides of the equality with respect to β, we obtain

$$\frac{\partial J}{\partial \beta} = -\frac{i}{2\pi}\int_{-\infty}^{\infty} e^{-iu\beta}\frac{d\overset{\smile}{E}(u)}{du}du = \frac{\beta}{2\pi}\int_{-\infty}^{\infty} e^{-iu\beta}\overset{\smile}{E}(u)du = \beta \cdot f_w(\beta)$$, (2.71)

where $f_w(\beta)$ – probability density of the random variable $W(t)$, taken at the argument β.

Integrating (2.71) from $-\infty$ to the current value of β, we obtain

$$J(\beta) = \int_{-\infty}^{\beta} wf_w(w)dw + c$$. (2.72)

To determine the constant of integration c, we substitute (2.72) into (2.69) and set $\beta = \infty$. Since in this case

$$\overline{e} \cdot \overline{v}_\beta = 0$$

(the probability of an emission at a very high level tends to zero), we obtain

$$c = -\frac{1}{2}\overline{w}$$. (2.73)

So,

$$\overline{e} \cdot \overline{v}_\beta = \overline{w} - \int_{-\infty}^{\beta} wf_w(w)dw - \beta \cdot \overline{\tau}_\beta \cdot \overline{v}_\beta$$,

that is, the average electric energy $\overline{e}$, which is limited by the realization of a random function above a given level β during the emission, if we consider the random function $W(t)$ to be stationary, is equal to

$$\overline{e} = \frac{\overline{w}}{\overline{v}_\beta} - \frac{1}{\overline{v}_\beta}\int_{-\infty}^{\beta} wf_w(w)dw - \beta \cdot \overline{\tau}_\beta$$. (2.74)

For further calculations, it is necessary to specify the form of the distribution law of the ordinates of the random function $W(t)$. If the process is normal, then

$$f_w(w) = \frac{1}{\sigma_w \sqrt{2\pi}} e^{-\frac{(w-\overline{w})^2}{2\sigma_w^2}} \qquad (2.75)$$

and the integral in (2.74) can be calculated. Performing the calculations, we obtain, taking into account (2.57), for the average emission electricity

$$\overline{e} = \frac{\sigma_w^2 \sqrt{2\pi}}{\sigma_v} + \frac{(\overline{w} - \beta)\sigma_w \pi}{\sigma_v} \left[1 - \Phi\left(\frac{\beta - \overline{w}}{\sigma_w} \right) \right] e^{\frac{(\beta-\overline{w})^2}{2\sigma_w^2}} . \qquad (2.76)$$

where $\sigma_w^2 = K_w(0)$, $\sigma_v^2 = -\frac{d^2}{d\tau^2} K_w(\tau)\Big|_{\tau=0}$,

$K_w(\tau)-$ autocorrelation power function $W(t)$.

To find the statistical characteristics of realizations of random processes, it is natural to use the principles of processing research material developed in mathematical statistics. To do this, it is necessary to proceed to the sequence of realizations of a random variable obtained by discretization of a random process.

$$w_1, w_2, \ldots w_i, \ldots, w_n , \qquad (2.77)$$

where $w_i = w(i \cdot \Delta), \quad i = 1, 2, \ldots, n, \quad n \cdot \Delta = T$.

Then the estimate of the mathematical expectation, that is, the mean, is found by the formula

$$\overline{w} = \frac{1}{n} \sum_{i=1}^{n} w_i . \qquad (2.78)$$

The sample variance is given by the formula

$$\sigma_w^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(w_i - \overline{w} \right)^2 , \qquad (2.79)$$

In turn, the sample estimate of the autocorrelation function is found by the formula

$$\tilde{R}_w(k) = \frac{1}{\sigma_w^2 (n-1)} \sum_{i=1}^{n-k} \left(w_i - \overline{w} \right)\left(w_{i+k} - \overline{w} \right), \quad k = 0, 1, \ldots, m . \qquad (2.80)$$

Considering the features of the stationary random process of the power of electricity consumption $W(t)$, it is advisable to choose the structure of the analytical formula of the autocorrelation function in the form [40]

$$R_w(\gamma,\tau) = e^{-\gamma\cdot|\tau|}\cdot\left(1+\gamma\cdot|\tau|\right), \qquad (2.81)$$

where γ - parameter.

To find the parameter γ of the analytical formula (2.81), it is necessary to approximate the formula (2.80) by minimizing the functional

$$F(\gamma) = \sum_{k=1}^{m}\left(R_w(\gamma,k\cdot\Delta) - \tilde{R}_w(k)\right)^2 \to \min_{\gamma}. \qquad (2.82)$$

Taking into account that

$$F(\gamma_0) = \min_{\gamma}\sum_{k=1}^{m}\left(R_w(\gamma,k\cdot\Delta) - \tilde{R}_w(k)\right)^2,$$

the analytical formula of the autocovariance function is written as

$$K_w(\gamma_0,\tau) = \sigma_w^2\cdot R_w(\gamma_0,\tau). \qquad (2.83)$$

Then, according to (2.83), we have

$$K_w(\gamma_0,0) = \sigma_w^2\cdot R_w(\gamma_0,0) = \sigma_w^2. \qquad (2.84)$$

Next, we find the variance of the random function $V(t)$ by calculating the second-order derivative of the autocovariance function (2.83)

$$\sigma_v^2 = -\sigma_w^2\cdot\left[-\gamma_0^2 e^{-\gamma_0\cdot\tau}\left(1-\gamma_0\tau\right)\right]\Big|_{\tau=0} = \gamma_0^2\cdot\sigma_w^2. \qquad (2.85)$$

Taking into account (2.85), formula (2.76) of the average emission electricity at a stationary random power of electricity W above a given level β will take the form

$$\overline{e} = \frac{\sigma_w\sqrt{2\pi}}{\gamma_0} - \frac{(\beta-\overline{w})\cdot\pi}{\gamma_0}\left[1-\Phi\left(\frac{\beta-\overline{w}}{\sigma_w}\right)\right]e^{\frac{1}{2}\left(\frac{\beta-\overline{w}}{\sigma_w}\right)^2}. \qquad (2.86)$$

Analysis of formula (2.86) shows that it depends on four independent arguments – $\beta,\gamma_0,\overline{w},\sigma_w$. Such a number of variables causes certain difficulties in studying the dependence of formula (2.89) on these separate arguments. Therefore, it seems advisable to combine these arguments into groups in the form of factors. For this purpose, two groups of factors are distinguished

$$\Delta = \frac{\beta - \overline{w}}{\sigma_w}, \quad \lambda = \sqrt{2\pi}\,\frac{\sigma_w}{\gamma_0} \tag{2.87}$$

The first group of multipliers determines the deviation of the given level of consumed electricity power β from the average value of consumed electricity $\overline{w}$, which is calculated in units of the root mean square deviation of consumed electricity power σ_w. The second group of multipliers is equal to the amount of electricity consumed at zero deviation of the given power level of consumed electricity β from the average value of consumed electricity $\overline{w}$, i.e., when Δ =0.

As a result of applying the notations (2.87), formula (2.86) will take the form

$$\overline{e}(\Delta,\lambda) = \lambda\left\{1 - \Delta\cdot\sqrt{\frac{\pi}{2}}\left[1-\Phi(\Delta)\right]e^{\frac{\Delta^2}{2}}\right\}. \tag{2.88}$$

Analysis of formula (2.88) shows that it depends on only two variables, which greatly simplifies its study. Moreover, formula (2.88) can be represented in a dimensionless form

$$\hat{e}(\Delta) = 1 - \Delta\cdot\sqrt{\frac{\pi}{2}}\left[1-\Phi(\Delta)\right]e^{\frac{\Delta^2}{2}}, \tag{2.89}$$

where $\hat{e}(\Delta) = \dfrac{\overline{e}(\Delta,\lambda)}{\lambda}$.

It should be emphasized that the study of formulas (2.88) or (2.89) using groups of factors (2.87) allows for one value of the factor to determine an infinite number of values of the factors that form this factor. It is clear that this greatly simplifies the study of the given formulas.

Fig. 2.1 shows the graph of the function (2.89).

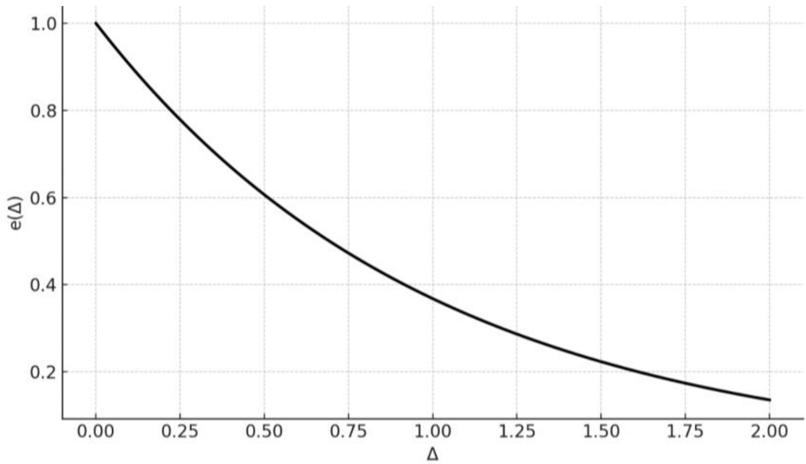

Figure 2.1. Graph of function (2.89)

Analysis of the graph shown in Fig. 2.1 shows that there is a monotonic decrease in the average value of electricity output. It is clear that the largest value of the average electricity output will be when $\Delta = 0$, i.e.

$$\overline{e}(\Delta = 0, \lambda) = \lambda, \qquad (2.90)$$

The graph of the function (2.89) shows that with an increase in the deviation from the average value of the electric power, the quality of the electric power supply monotonically decreases.

It seems advisable to approximate the obtained formula (2.89) for ease of use with a simpler analytical expression. Analysis of the graph of the function (2.89) indicates the convenience of choosing the structure of the formula in the form of an exponential dependence, i.e.

$$\hat{e}(\Delta) = e^{-k \cdot \Delta}, \qquad (2.91)$$

where $k -$ parameter.

To find the parameter, the functionality was applied

$$S(k) = \sum_{i=1}^{n} \left(1 - \Delta_i \cdot \sqrt{\frac{\pi}{2}} \left[1 - \Phi(\Delta_i) \right] e^{\frac{\Delta_i^2}{2}} - e^{-k \cdot \Delta_i} \right)^2 \quad i = 1, 2, \ldots, n, \quad n = 20 \qquad (2.92)$$

Minimization of the functional (2.92) with respect to the parameter k gave the optimal value of the parameter

$$k_0 = -1.115 \quad (2.93)$$

При цьому коефіцієнт кореляції склав величину

$$r = 0.999 \quad (2.94)$$

According to the Chaddock scale [39], such a value of the correlation coefficient indicates a very high relationship between the functions under study.

Taking into account (2.93), the approximation of model (2.91) will take the form

$$\hat{e}(\Delta) = e^{-1.115\cdot\Delta} \quad (2.95)$$

Fig. 2.2 presents the graphs of functions (2.89) and (2.95). Analysis of the graphs shown in Fig. 2.2 indicates a fairly good approximation of the model represented by function (2.89) to formula (2.95). Thus, in the future it is advisable to use formula (2.95) instead of formula (2.89). Then formula (2.88) will take the form

$$\overline{e}(\Delta, \lambda) = \lambda \cdot e^{-1.115\cdot\Delta} \quad (2.96)$$

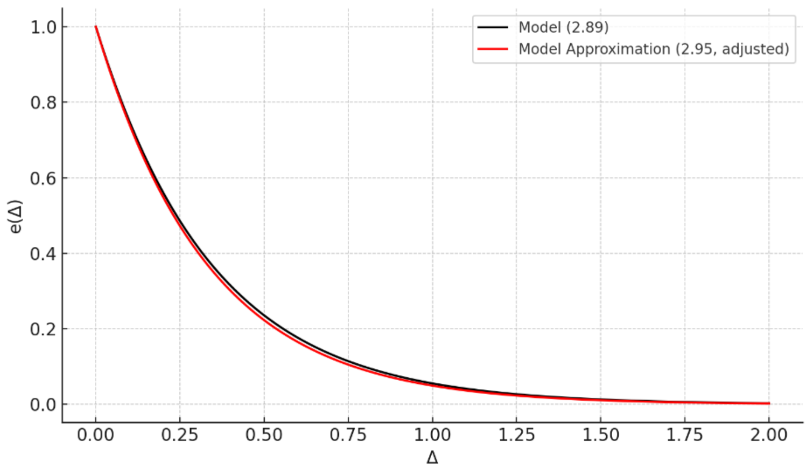

Figure 2.2 Power levels of electricity consumption

All further calculations and construction of tables and graphs were carried out using Excel spreadsheets and the Mathcad mathematical package. As an example for greater clarity, let us consider the electricity consumption at an operating underground mine - mine No. 9, as the most typical in its industry (Table 2.1).

Table 2.1

Data on power consumption of electricity

№	Year	Power consumption (kW)
0	2001	10200
1	2002	10250
2	2003	10000
3	2004	10200
4	2005	10300
5	2006	10250
6	2007	10110
7	2008	10250
8	2009	10300
9	2010	10200
10	2011	10250
11	2012	10245
12	2013	10242
13	2014	10445
14	2015	10500
15	2016	10250
16	2017	10300
17	2018	10400
18	2019	10100
19	2020	10500

Fig. 2.2 shows a graph of electricity consumption capacity according to the data in Table 2.2.

Analysis of the graph shown in Fig. 2.2 shows that the deviation of the power consumption of electricity is stochastic in nature relative to the average value of the power consumption of electricity, the graph of which is marked in red. The average value of the power consumed by electricity is equal to

$$\overline{w} = \frac{1}{20}\sum_{i=0}^{19} w_i = 10264.6 \quad , \qquad (2.96)$$

The standard deviation of the consumed electricity is equal to

$$\sigma_w = \sqrt{\frac{1}{19}\sum_{i=0}^{19}\left(w_i - 10264.6\right)^2} = 125.86 \quad (2.97)$$

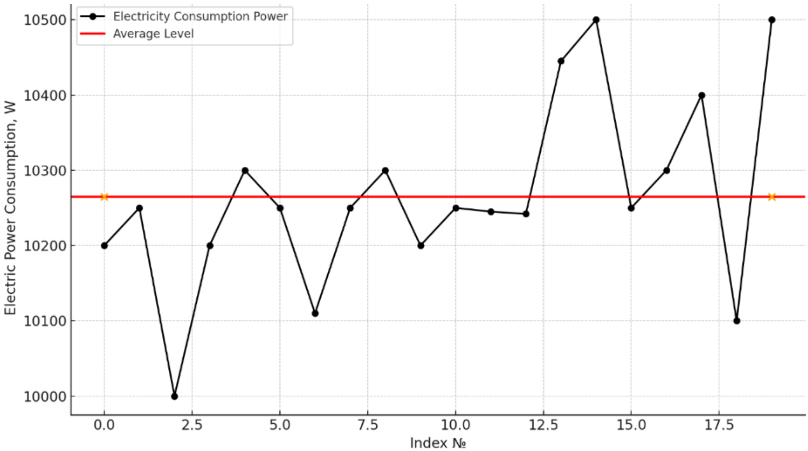

Figure 2.2. Electricity consumption power graph

The sample estimate of the autocorrelation function is found, according to (2.14), by the formula

$$\tilde{R}_w(k) = \frac{1}{300950.8}\sum_{i=1}^{n-k}\left(w_i - 10264.6\right)\left(w_{i+k} - 10264.6\right), \quad k = 0,1,\ldots,5 \quad (2.98)$$

Formula (2.81) was chosen as the structure of the analytical formula for the autocorrelation function of power $W(t)$.

To approximate functions (2.81) and (2.98) using the parameter γ, the minimization of functional (2.82) was applied. As a result, we obtain that $\gamma_0 = 3.646$, and formula (2.81) takes the form

$$R_w(\tau) = e^{-3,646\cdot|\tau|}\left(1 + 3,646\cdot|\tau|\right) \quad (2.99)$$

Fig. 2.3 presents graphs of the sample estimate of the autocorrelation function (2.98) and its approximation (2.99).

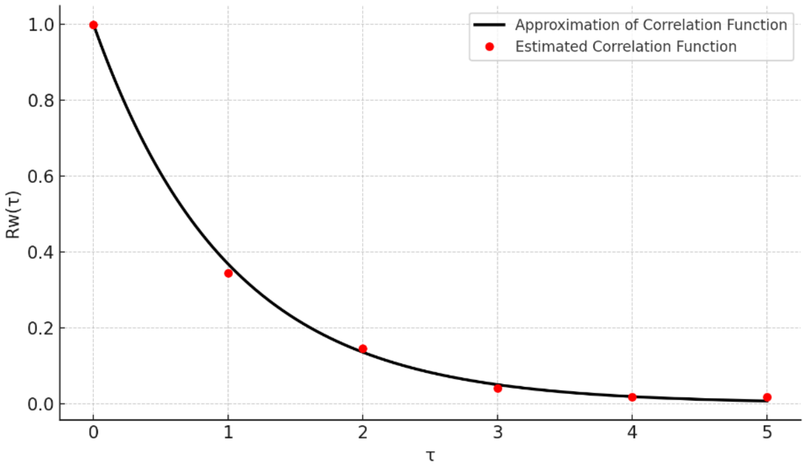

Figure 2.3. Graphs of the sample estimate of the correlation function (2.28) and its approximation (2.99)

Analysis of the graphs shown in Fig. 2.3 shows a satisfactory approximation of the sample estimate of the correlation function (2.98) by its approximation (2.99) within the specified time shift limits.

Next, we calculate the correlation coefficient to determine the reliability of formula (2.99). The calculation of the coefficient of determination was carried out by the formula

$$r = \sqrt{1 - \frac{ESS}{TSS}}, \tag{2.100}$$

where $ESS = \sum_{k=0}^{5} \left[\tilde{R}_w(k) - R_w(k) \right]^2$,

$$TSS = \sum_{k=0}^{5} \left[\tilde{R}_w(k) - \frac{1}{6} \sum_{k=0}^{5} \tilde{R}_w(k) \right]^2.$$

Calculating the necessary amounts and substituting them into formula (2.100), we find

$$r = \sqrt{1 - \frac{0.013273}{0.732584}} = 0.991 \quad (2.101)$$

According to the Chaddock scale, the value of the coefficient of determination (2.101) is above 0.9, i.e.

$$r = 0.991 > 0.9 ,$$

which indicates a "very high" closeness of the connection in absolute value.

According to formulas (2.83) and (2.99), the analytical formula of the autocovariance function takes the form

$$K_w(\tau) = 15839.5 \cdot e^{-3.648 \cdot |\tau|} \left(1 + 3.646 \cdot |\tau|\right) . \quad (2.102)$$

Using formula (2.19), we have

$$\sigma_v = 3.646 \cdot 125.86 = 458.87 . \quad (2.103)$$

Using the above formulas, we find the qualitative characteristics of electricity use at the studied mine.

According to formula (2.57) taking into account (2.96), (2.97), (2.103), a formula was obtained that determines the average number of emissions per unit of time (in our case per year) relative to the average power of consumed electricity (2.96)

$$\bar{v}_\beta = \frac{\gamma_0}{2\pi} e^{-\frac{(\beta - \bar{w})^2}{2\sigma_w^2}} ,$$

or, taking into account the numerical values of the quantities

$$\bar{v}_\beta = 0.58 \cdot e^{-\frac{(\beta - 10264.6)^2}{31679.03}} . \quad (2.104)$$

Fig. 2.4 shows a graph of the average annual emissions versus the emission limit. Analysis of the graph shows that as the emission limit increases, the average annual emissions decrease quite rapidly.

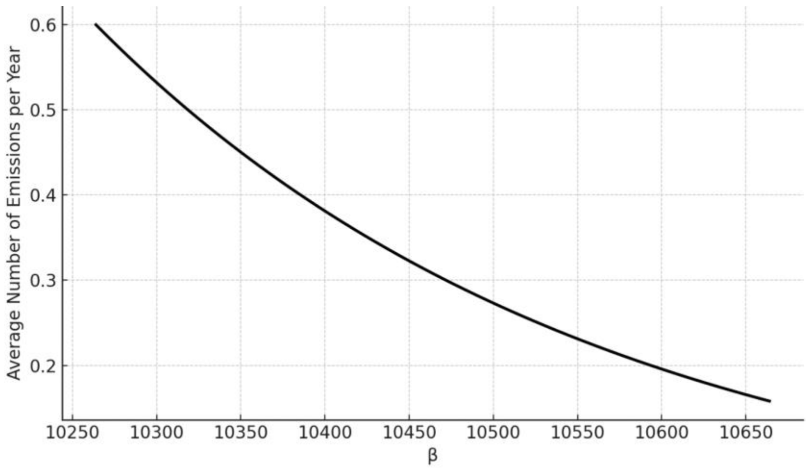

Figure 2.4. Graph of the dependence of the average amount of electricity emissions per year on the limit of the amount of electricity emissions relative to the average amount of electricity consumed

According to formula (2.58) taking into account (2.96), (2.97), (2.103), a formula was obtained that determines the average duration of emissions per unit of time (in our case per year) relative to the emission limit (2.96)

$$\bar{\tau}_\beta = \frac{\pi}{\gamma_0} e^{-\frac{(\beta-\bar{w})^2}{2\sigma_w^2}} \left[1-\Phi\left(\frac{\beta-\bar{w}}{\sigma_w}\right)\right],$$

or taking into account the numerical values of the quantities

$$\bar{\tau}_\beta = 0.8617 e^{-\frac{(\beta-10264.6)^2}{31679.08}} \left[1-\Phi\left(\frac{\beta-10264.6}{125.86}\right)\right]. \quad (2.105)$$

Fig. 2.5 shows a graph of the average duration of emissions per year depending on the value of the emission limit. Analysis of the graph shows that with an increase in the value of the emission limit, the average duration of emissions per year decreases quite rapidly.

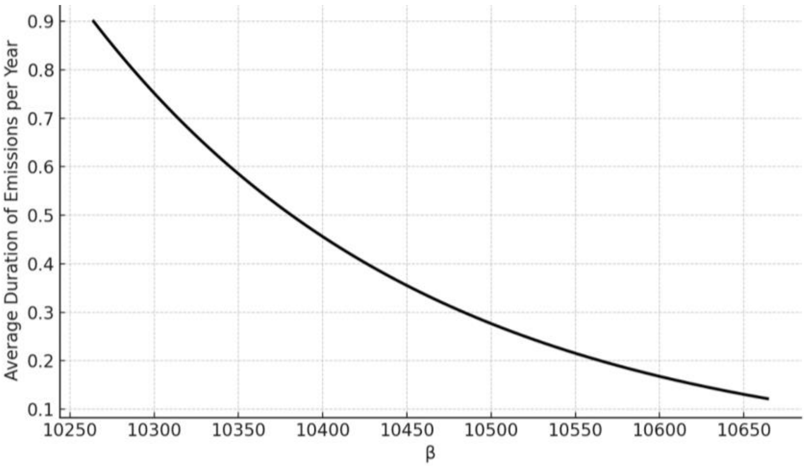

Figure 2.5. Graph of the dependence of the average duration of electricity emissions per year on the limit of the amount of electricity emissions relative to the average power of electricity consumed

An important assessment of the quality of electricity consumption is the average value of the electricity output at a stationary random electricity power W above a given level β, which is determined by the formula (2.86)

$$\overline{e}_{\beta} = \frac{\sigma_w \sqrt{2\pi}}{\gamma_0} - \frac{(\beta - \overline{w}) \cdot \pi}{\gamma_0} \left[1 - \Phi\left(\frac{\beta - \overline{w}}{\sigma_w} \right) \right] \cdot e^{\frac{(\beta - \overline{w})^2}{2\sigma_w^2}},$$

or taking into account the numerical values of the quantities

$$\overline{e}_{\beta} = 86.529 - 0.862 \cdot (\beta - 10264.6) \cdot \left[1 - \Phi\left(\frac{\beta - 10264.6}{125.86} \right) \right] \cdot e^{\frac{(\beta - 10264.6)^2}{31679.08}}. \quad (2.106)$$

Fig. 2.6 shows a graph of the average annual electricity output versus the emission limit. Analysis of the graph shows that as the emission limit increases, the average annual electricity output decreases quite rapidly.

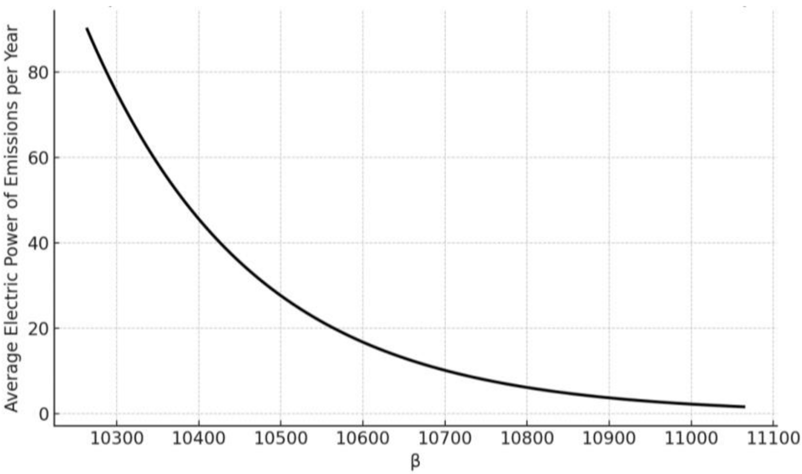

Figure 2.6. Graph of the dependence of the average annual power output on the limit of the power output relative to the average power of consumed electricity

Thus, the calculations show that during the selected period of electricity consumption at mine No. 9 (Kryvyi Rih), random emissions of electricity consumption occurred due to statistical features of the power consumption of electricity.

An example of applying the calculation of electricity supply at a mining enterprise confirmed the possibility of using the developed methodology.

It is advisable to direct further research towards studying the factors influencing EQI at these types of iron ore enterprises.

2.2. Synthesis of factor impact on power quality

It is clear that the quality of electricity is influenced by many factors. Moreover, the influence of these factors on the quality of electricity varies in strength. Thus, to build a mathematical multifactor model for the quality of electricity, it is necessary to take into account not only the factors themselves, but also the degree of influence of each factor on the overall quality of electricity. The formation of a multifactor model best reflects and integrates the influence of factors on the quality of electricity.

A multifactor model of electricity quality is understood as a dependence F that connects the total electricity quality Q with its input $\vec{q} = (q_1, q_2, \dots, q_n)$ – the vector of electricity qualities of individual components

$$Q = F(\vec{q}), \tag{2.107}$$

According to the general approach to building a mathematical model of the object under consideration, the first step is to conduct a structural synthesis of the model. At this stage, the type of dependence F is determined without taking into account the values of its parameters. Let us conditionally perform the following operation: "split" the model F into its structure St and parameters $c_1, c_2, \dots, c_n$, that is, present the model in the form of a pair

$$F = \left\langle St, \vec{C} \right\rangle, \tag{2.108}$$

where $\vec{C} = (c_1, c_2, \dots, c_n)$ – vector of model parameters.

At the stage of structural synthesis, only the structure St of the model is determined, and the specific values of the parameters of the vector $\vec{C}$ are not of interest. In general, structure is understood as the type of elements that make up the object and the relationships between the elements. There are many different structures of mathematical models. Linearity, staticity, determinism, discreteness, multiplicativity, etc. are structural categories [39].

Analysis of the quality of electricity Q as a multifactor model depending on the magnitude of the qualities of electricity of its components $q_i, \quad (i = 1, 2, \dots, n)$ shows that the structure of this model can be represented as static and multiplicative.

In this case, the model is written in the form

$$Q = k \cdot q_1^{c_1} \cdot q_2^{c_2} \cdot \ldots \cdot q_n^{c_n}, \tag{2.109}$$

or in a rolled up form

$$Q = k \cdot \prod_{i=1}^{n} q_i^{c_i}, \tag{2.110}$$

The specific values of the parameters $c_1, c_2, \ldots, c_n$ are not yet important, only the type of dependence, i.e. the multiplicativeness of the structure St, is important.

Thus, at the stage of structural synthesis, only the type and nature of the model is determined, and its parameters are determined at the stage of identification of the model parameters. At the same time, it makes sense to analyze the properties of the selected structure of the multifactor model (3). According to the definition of electricity quality, there are restrictions

$$0 \le Q \le 1, \quad 0 \le q_i \le 1, \quad (i = 1, 2..., n). \tag{2.111}$$

Moreover, let

$$\hat{q}_i = 1, \quad (i = 1, 2, ..., n), \tag{2.112}$$

then

$$Q = \lim_{n\to\infty} k \cdot \prod_{i=1}^{n} \hat{q}_i^{c_i} = k \cdot \lim_{n\to\infty} \prod_{i=1}^{n} \hat{q}_i^{c_i} = k \cdot 1 = k. \tag{2.113}$$

But under condition (2.112) according to (2.113) we have

$$Q = 1. \tag{2.114}$$

Taking into account (2.112) and (2.113), we have

$$k = 1. \tag{2.115}$$

According to (2.115), the mathematical model (2.110) will take the form

$$Q = \prod_{i=1}^{n} q_i^{c_i}. \tag{2.116}$$

The obtained structure of the model (2.116) meets all the requirements of (2.111).

Let us determine the content of the parameters $c_1, c_2, \ldots, c_n$ in the model (2.116). If the parameter $c_i = 0$, then the component q_i, which determines the local quality of electricity, is not taken into account.

To find out the content of the parameters c_i, we prologarithmize the formula (2.116)

$$\ln Q = \ln\left(\prod_{i=1}^{n} q_i^{c_i}\right), \quad \ln Q = \sum_{i=1}^{n} c_i \cdot \ln q_i . \tag{2.117}$$

In formula (2.117), we calculate the partial derivative with respect to the variable q_i

$$\frac{\partial \ln Q}{\partial q_i} = \frac{\partial}{\partial q_i}\left(\sum_{i=1}^{n} c_i \cdot \ln q_i\right) = \frac{c_i}{q_i}, \quad \frac{1}{Q}\frac{\partial Q}{\partial q_i} = \frac{c_i}{q_i} . \tag{2.118}$$

Thus, according to (2.118), the value of the parameter c_i determines the sensitivity of the overall quality Q to the quality component q_i, i.e. how strongly the value of the quality component q_i affects the value of the overall quality Q. It is clear that the constraint must also be satisfied

$$c_i \geq 0, \quad (i = 1, 2, ..., n) . \tag{2.119}$$

The next stage of model synthesis is to identify the parameters of the model c_i, which is associated with determining the numerical values of the parameters $\vec{C} = (c_1, c_2, ..., c_n)$. The initial information for identification is the structure St and observation of the behavior of the input $\vec{q}(t)$ and output $Q(t)$ of the object in real conditions. Thus, the pair

$$I(t) = \left\langle \vec{q}(t); Q(t) \right\rangle \tag{2.120}$$

In general, it is the main source of information for identification.

Considering that the observations of the states of input $\vec{q}(t)$ and output $Q(t)$ during operation are carried out at discrete moments of time, formula (120) takes the form

$$I = \left\langle \vec{q}_k; Q_k \right\rangle, \quad k = 1, 2, ..., N, \tag{2.121}$$

where k – number of time points t_k when the values of $\hat{q}(t)$ and $Q(t)$ were recorded, i.e. $\hat{q}_k = \hat{q}(t_k)$, $Q_k = Q(t_k)$. Thus, the initial data required for identification is formed by two

$$\langle St, I \rangle, \tag{2.122}$$

that is, the structure of the model (2.116) and the observations (2.120).

The process of determining the parameters of the model (2.116) is reduced to determining the parameters $\hat{C} = (c_1, c_2, ..., c_n)$ from the initial data (2.122), i.e.

$$\hat{C} = \varphi(St, I), \tag{2.123}$$

where φ – an identification algorithm that determines how to find the parameters $\hat{C} = (c_1, c_2, ..., c_n)$, knowing St and I.

In practice, two types of identification algorithms are used: adaptive and non-adaptive. An adaptive identification algorithm is an algorithm that allows you to refine the values of the model parameters being identified as additional information is obtained. In contrast to the adaptive identification algorithm, a non-adaptive algorithm allows you to obtain the desired parameters $\hat{C} = (c_1, c_2, ..., c_n)$ at once, using all the information (120), and not by gradually refining them.

Given that the static model (2.116) is considered, it is advisable to use a non-adaptive identification algorithm to find the values of the parameters. In this case, the initial data take the form (2.120). According to the non-adaptive identification algorithm, it is necessary to substitute the data (2.120) into the formula (2.116). As a result, we obtain a system of N equations with n unknowns

$$q_{1,1}^{c_1} \cdot q_{2,1}^{c_2} \cdot \ldots \cdot q_{n,1}^{c_n} = Q_1,$$

$$q_{1,2}^{c_1} \cdot q_{2,2}^{c_2} \cdot \ldots \cdot q_{n,2}^{c_n} = Q_2,$$

…………………….

$$q_{1,N}^{c_1} \cdot q_{2,N}^{c_2} \cdot \ldots \cdot q_{n,N}^{c_n} = Q_N,$$

which is conveniently written in the form

$$\prod_{i=1}^{n} q_{i,k}^{c_i} = Q_k , \quad (k = 1, 2, ..., N) . \qquad (2.124)$$

It is clear that in this case it is necessary to fulfill the condition $N \geq n$. Otherwise, the number of unknowns will be less than the number of equations and the system (2.124) will not have a unique solution.

Thus, the problem of non-adaptive identification of a static model (2.116) is reduced to solving the system of equations (2.124). This system has two essential properties that determine the difficulties that arise when solving it: incompatibility and transcendence. Its incompatibility is associated with the case $N > n$ (the number of equations is greater than the number of unknowns), and transcendence is associated with an arbitrary choice of the model structure. It is obvious that the problems of the non-adaptive method of solving the identification problem are associated with solving an incompatible transcendental system of equations. Let us start with incompatibility, i.e. when $N > n$.

We solve the inconsistent system of equations (2.124) by the method of least squares, i.e., by minimizing the total residual of the right and left sides of the equations of this system. To do this, we form the total residual function in the form of the sum of the squares of the residuals of each of the equations

$$F(c_1, c_2, ..., c_n) = \sum_{k=1}^{N} \left[\prod_{i=1}^{n} q_{i,k}^{c_i} - Q_k \right]^2 . \qquad (2.125)$$

The function (2.125) is non-negative and equals zero when the right and left sides of the equations of the system (2.124) coincide. In this case, the closer the right sides of the equation of the system (2.124) are to the left sides, the smaller the value of the residual function (2.125). This gives grounds to consider the following values of the parameters as solutions of the system (2.124)

$$\vec{C}^* = (c^*_1, c^*_2, ..., c^*_n) , \qquad (2.126)$$

at which the residual function (2.19) is minimal, i.e.

$$F(c^*_1, c^*_2, ..., c^*_n) = \min_{c_1, c_2, ..., c_n} F(c_1, c_2, ..., c_n) .$$

Therefore, to solve the inconsistent system of equations (2.124), it is sufficient to minimize the total residual function (2.125).

For the identification of the function (2.124), the total residual function (2.125) is quite complicated, since it includes exponential functions. Therefore, it is proposed to apply such a transformation that would convert the multiplication operation into the addition operation without changing the result of minimizing the function (2.124). For this, it is possible to apply the action of logarithmization, which converts multiplication into addition. As a result, formula (2.116) will take the form (2.110)

$$\ln Q = \sum_{i=1}^{n} c_i \cdot \ln q_i .$$

For the convenience of further calculations, it is advisable to introduce the notation

$$\ln Q = y, \quad \ln q_i = x_i . \qquad (2.127)$$

Then formula (2.111) taking into account (2.21) will take the form

$$y = \sum_{i=1}^{n} c_i \cdot x_i . \qquad (2.128)$$

The specificity of such a structure lies in the linearity of the function with respect to the parameters being determined , $c_1, c_2, \ldots, c_n$.

In turn, the total residual function will be written as

$$\delta(c_1, c_2, \ldots, c_n) = \sum_{k=1}^{N} \left(\sum_{i=1}^{n} c_i \cdot x_{i,k} - y_k \right)^2 . \qquad (2.129)$$

This linearity allows us to reduce the problem of minimizing the residual function (2.129) to solving a system of linear algebraic equations.

The simple form of the function (2.128) allows us to solve the problem of minimizing (2.129) by equating the partial derivatives of the function (2.129) to zero, in accordance with the necessary condition for the existence of an extremum, i.e.

$$\frac{\partial}{\partial c_j} \delta(c_1, c_2, \ldots, c_n) = \frac{\partial}{\partial c_j} \left[\sum_{k=1}^{N} \left(\sum_{i=1}^{n} c_i \cdot x_{i,k} - y_k \right)^2 \right] = 0, \; (j = 1, 2, \ldots, n) \qquad (2.130)$$

Since the function (2.128) is a linear function with respect to the parameters $c_1, c_2, \ldots, c_n$, then $\delta(c_1, c_2, \ldots, c_n)$ in (2.129) is a quadratic function, which determines the linearity of the system of equations (2.130). Indeed, calculating the partial derivative in (2.130), we have

$$\frac{\partial}{\partial c_j}\left[\sum_{k=1}^{N}\left(\sum_{i=1}^{n} c_i \cdot x_{i,k} - y_k\right)^2\right] = 2\sum_{k=1}^{N}\left(\sum_{i=1}^{n} c_i \cdot x_{i,k} - y_k\right)\cdot x_{j,k} = 0,$$

$$\sum_{k=1}^{N}\left(\sum_{i=1}^{n} c_i \cdot x_{i,k} - y_k\right)\cdot x_{j,k} = \sum_{i=1}^{n} c_i \cdot \varphi_{i,j} - \eta_j = 0, \qquad (2.131)$$

where $\varphi_{i,j} = \sum_{k=1}^{N} x_{i,k} \cdot x_{j,k}$, $\eta_j = \sum_{k=1}^{N} y_k \cdot x_{j,k}$, $(j,i = 1,2,...,n)$.

As can be seen, (2.131) is a system of linear algebra equations with respect to the identified parameters $c_1, c_2, ..., c_n$,

$$\begin{cases} c_1 \cdot \varphi_{1,1} + c_2 \cdot \varphi_{2,1} + ... + c_n \cdot \varphi_{n,1} = \eta_1 \\ c_1 \cdot \varphi_{1,2} + c_2 \cdot \varphi_{2,2} + ... + c_n \cdot \varphi_{n,2} = \eta_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ c_1 \cdot \varphi_{1,n} + c_2 \cdot \varphi_{2,n} + ... + c_n \cdot \varphi_{n,n} = \eta_n \end{cases}, \qquad (2.132)$$

which is solved by standard methods.

To investigate the solution of the system of equations (2.132), we consider the matrix of this system of coefficients with unknown parameters $c_1, c_2, ..., c_n$ of size $(n \times n)$

$$\Phi = \begin{Vmatrix} \varphi_{1,1} & \varphi_{2,1} & \cdots & \varphi_{n,1} \\ \varphi_{1,2} & \varphi_{2,2} & \cdots & \varphi_{n,2} \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{1,n} & \varphi_{2,n} & \cdots & \varphi_{n,n} \end{Vmatrix}. \qquad (2.133)$$

This matrix is called the Fisher information matrix [36, 37]. It is symmetric because $\varphi_{i,j} = \varphi_{j.i}$. For a unique solution of system (2.132) it is necessary that the determinant of this matrix is not equal to zero, i.e.

$$|\Phi| = \begin{vmatrix} \varphi_{1,1} & \varphi_{2,1} & \cdots & \varphi_{n,1} \\ \varphi_{1,2} & \varphi_{2,2} & \cdots & \varphi_{n,2} \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{1,n} & \varphi_{2,n} & \cdots & \varphi_{n,n} \end{vmatrix} \neq 0 . \qquad (2.134)$$

This determinant is zero in two cases: when $n > N$, i.e. when there are not enough measurements, and at $x_1, x_2, \ldots, x_n$, i.e. when more than $N - n$ states are linearly dependent. There are two ways to respond to this situation: either increase the number of observations N, or decrease the number n.

Let us consider Cramer's method for solving the system of algebraic equations (2.132) [41].

For this purpose, Cramer's formulas are used. First, the determinant is constructed according to the system of equations (2.132) from the coefficients for the unknown parameters $c_1, c_2, \ldots, c_n$

$$\Delta = \begin{vmatrix} \varphi_{1,1} & \varphi_{2,1} & \cdots & \varphi_{n,1} \\ \varphi_{1,2} & \varphi_{2,2} & \cdots & \varphi_{n,2} \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{1,n} & \varphi_{2,n} & \cdots & \varphi_{n,n} \end{vmatrix} \qquad (2.135)$$

If $\Delta \neq 0$, then we compose the determinants, successively replacing the columns in (2.135) with the columns of the free terms of the system (2.135)

$$\Delta_1 = \begin{vmatrix} \eta_1 & \varphi_{2,1} & \cdots & \varphi_{n,1} \\ \eta_2 & \varphi_{2,2} & \cdots & \varphi_{n,2} \\ \cdots & \cdots & \cdots & \cdots \\ \eta_n & \varphi_{2,n} & \cdots & \varphi_{n,n} \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} \varphi_{1,1} & \eta_1 & \cdots & \varphi_{n,1} \\ \varphi_{1,2} & \eta_2 & \cdots & \varphi_{n,2} \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{1,n} & \eta_n & \cdots & \varphi_{n,n} \end{vmatrix}, \ldots,$$

$$\Delta_n = \begin{vmatrix} \varphi_{1,1} & \varphi_{2,1} & \cdots & \eta_1 \\ \varphi_{1,2} & \varphi_{2,2} & \cdots & \eta_2 \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{1,n} & \varphi_{2,n} & \cdots & \eta_n \end{vmatrix} . \qquad (2.136)$$

The solution of system (2.132) is found by Cramer's formula by successively dividing (2.134) by (2.133)

$$c_1^{\times} = \frac{\Delta_1}{\Delta}, \quad c_2^{\times} = \frac{\Delta_2}{\Delta}, \ \ldots, \quad c_n^{\times} = \frac{\Delta_n}{\Delta} . \qquad (2.137)$$

Thus, the parameter values (2.137) minimize the residual function (2.129), i.e.

$$S(c_1^{\times}, c_2^{\times}, \ldots, c_n^{\times}) = \min_{c_1, c_2, \ldots, c_n} S(c_1, c_2, \ldots, c_n) .$$

Moreover, these values of parameters (2.136) coincide with parameters (2.126), i.e. they also minimize function (2.125), i.e. the following holds:

$$F(c_1^{\times}, c_2^{\times}, \ldots, c_n^{\times}) = \min_{c_1, c_2, \ldots, c_n} F(c_1, c_2, \ldots, c_n) .$$

The multifactor mathematical model for electricity quality according to (2.116) and (2.135) will take the form

$$Q_m = \prod_{i=1}^{n} q_i^{c_i^{\times}} . \qquad (2.138)$$

Model (2.138) allows us to estimate the relative error in determining the quality of electricity based on the known errors in the qualities of its components. Indeed, we take the logarithm of (2.138) and calculate the differential

$$\ln Q_m = \sum_{i=1}^{n} c_i^{\times} \cdot \ln q_i ,$$

$$d(\ln Q_m) = d\left(\sum_{i=1}^{n} c_i^{\times} \cdot \ln q_i \right) ,$$

$$\frac{dQ_m}{Q_m} = \sum_{i=1}^{n} c_i^{\times} \cdot \frac{dq_i}{q_i}. \quad (2.139)$$

Assuming that

$$dQ_m \approx \Delta Q_m, \; dq_i \approx \Delta q_i, \quad (i = 1, 2, ..., n), \quad (2.140)$$

we get the desired formula

$$\delta Q_m = \sum_{i=1}^{n} c_i^{\times} \cdot \delta q_i, \quad (2.141)$$

where $\delta Q_m = \frac{\Delta Q_m}{Q_m}$, $\delta q_i = \frac{\Delta q_i}{q_i}, \quad (i = 1, 2, ..., n)$ – relative errors of electricity quality.

Let's consider building a multifactor model for electricity quality using a specific example.

Let three parameters of electricity quality be selected: voltage U, frequency ω, and the shape of the electric current curve χ. It is known that the voltage quality U is defined as q_1, the frequency quality ω – q_2, and the shape of the electric current curve χ – q_3. Then the multifactor model for electricity quality will be written in the form

$$Q = q_1^{c_1} \cdot q_2^{c_2} \cdot q_3^{c_3}. \quad (2.142)$$

To identify the model (2.142), i.e. to find the unknown parameters c_1, c_2, c_3, it is necessary to have statistical material regarding the qualities included in the model (2.142). The corresponding statistical material is presented in Table 3. According to the data in Table 3, the volume of statistics was 10 data. The first four columns contain the initial information regarding the general quality of electricity and the qualities of electricity of three parameters: voltage, frequency and shape of the electric current curve. The next four columns contain the results of logarithmic transformations of the initial data. As a result of logarithmic transformations of the initial data, the mathematical model took the form

$$y = c_1 \cdot x_1 + c_2 \cdot x_2 + c_3 \cdot x_3. \quad (2.143)$$

In this case, the total residual function will be written as

$$\delta(c_1, c_2, c_3) = \sum_{k=1}^{10} \left(c_1 \cdot x_{1,k} + c_2 \cdot x_{2,k} + c_3 \cdot x_{3,k} - y_k \right)^2 \quad (2.144)$$

According to the necessary minimization condition (2.38), we set the partial derivative with respect to the unknown parameter c_i to zero.

$$\frac{\partial}{\partial c_i} \delta(c_1, c_2, c_3) = 2 \sum_{k=1}^{10} \left(c_1 \cdot x_{1,k} + c_2 \cdot x_{2,k} + c_3 \cdot x_{3,k} - y_k \right) \cdot x_{i,k} = 0, \; (i = 1,2,3) \quad (2.145)$$

Table 2.3

Information on the identification of the three-factor model by power quality

№	Q	q_1	q_2	q_3	$y = \ln Q$	$x_1 = \ln q_1$	$x_2 = \ln q_2$	$x_3 = \ln q_3$
1	0.9	0.95	0.96	0.97	-0.105	-0.051	-0.041	-0.030
2	0.89	0.92	0.93	0.95	-0.117	-0.083	-0.073	-0.051
3	0.85	0.91	0.93	0.9	-0.163	-0.094	-0.073	-0.105
4	0.88	0.93	0.94	0.89	-0.128	-0.073	-0.062	-0.117
5	0.84	0.91	0.89	0.9	-0.174	-0.094	-0.117	-0.105
6	0.85	0.92	0.9	0.88	-0.163	-0.083	-0.105	-0.128
7	0.83	0.9	0.89	0.87	-0.186	-0.105	-0.117	-0.139
8	0.87	0.89	0.89	0.91	-0.139	-0.117	-0.117	-0.094
9	0.91	0.96	0.92	0.95	-0.094	-0.041	-0.083	-0.051
10	0.93	0.98	0.95	0.97	-0.073	-0.020	-0.051	-0.030

Expanding the condition (2.144), we obtain a system of three linear algebraic equations with three unknowns c_1, c_2, c_3

$$\begin{cases} c_1 \cdot \sum\limits_{k=1}^{10} x_{1,k}^2 + c_2 \cdot \sum\limits_{k=1}^{10} x_{1,k} x_{2,k} + c_3 \cdot \sum\limits_{k=1}^{10} x_{1,k} x_{3,k} = \sum\limits_{k=1}^{10} x_{1,k} y_k \\ c_1 \cdot \sum\limits_{k=1}^{10} x_{2,k} x_{1,k} + c_2 \cdot \sum\limits_{k=1}^{10} x_{2,k}^2 + c_3 \cdot \sum\limits_{k=1}^{10} x_{2,k} x_{3,k} = \sum\limits_{k=1}^{10} x_{2,k} y_k \\ c_1 \cdot \sum\limits_{k=1}^{10} x_{3,k} x_{1,k} + c_2 \cdot \sum\limits_{k=1}^{10} x_{3,k} x_{2,k} + c_3 \cdot \sum\limits_{k=1}^{10} x_{3,k}^2 = \sum\limits_{k=1}^{10} x_{3,k} y_k \end{cases} \quad (2.146)$$

The results of calculating the sums present in the system of equations (146) are presented in Table 4.

Table 2.4

Results of calculations of coefficients of the system of equations (2.146)

№	x_1^2	x_1x_2	x_1x_3	x_2^2	x_2x_3	x_3^2	x_1y	x_2y	x_3y
1	0.002631	0.002094	0.001562	0.001666	0.001243	0.000928	0.005404	0.004301	0.003209
2	0.006952	0.006051	0.004277	0.005267	0.003722	0.002631	0.009717	0.008457	0.005977
3	0.008895	0.006844	0.009937	0.005267	0.007646	0.011101	0.015327	0.011794	0.017123
4	0.005267	0.00449	0.008457	0.003829	0.007211	0.01358	0.009277	0.00791	0.014897
5	0.008895	0.01099	0.009937	0.01358	0.012278	0.011101	0.016443	0.020318	0.01837
6	0.006952	0.008785	0.010659	0.011101	0.013469	0.016341	0.013551	0.017123	0.020775
7	0.011101	0.012278	0.014673	0.01358	0.016229	0.019394	0.019632	0.021714	0.025949
8	0.01358	0.01358	0.01099	0.01358	0.01099	0.008895	0.016229	0.016229	0.013134
9	0.001666	0.003404	0.002094	0.006952	0.004277	0.002631	0.00385	0.007864	0.004838
10	0.000408	0.001036	0.000615	0.002631	0.001562	0.000928	0.001466	0.003722	0.00221
Сума	0.066347	0.069553	0.073201	0.077453	0.078628	0.087529	0.110896	0.119432	0.126482

According to the data in Table 2.2, the system of equations (2.146) takes the form

$$\begin{cases} 0.066347\cdot c_1+0.069553\cdot c_2+0.073201\cdot c_3=0.110896 \\ 0.069553\cdot c_1+0.077453\cdot c_2+0.078628\cdot c_3=0.119432 \\ 0.073201\cdot c_1+0.078628\cdot c_2+0.087529\cdot c_3=0.126482 \end{cases} \quad 2.(147)$$

The Fisher matrix of system (2.147) has the for

$$\Phi=\begin{Vmatrix} 0.066347 & 0.069553 & 0.073201 \\ 0.069553 & 0.077453 & 0.078628 \\ 0.073201 & 0.078628 & 0.087529 \end{Vmatrix}. \quad (2.148)$$

Since the determinant of the Fisher matrix (2.148) is not zero,

$$\left|\Phi\right|=1.8\cdot 10^{-6}\neq 0,$$

then the system of equations has a unique solution. Let us find this solution using Cramer's formulas [36, 37]. Let us find the determinants

$$\Delta=\begin{vmatrix} 0.066347 & 0.069553 & 0.073201 \\ 0.069553 & 0.077453 & 0.078628 \\ 0.073201 & 0.078628 & 0.087529 \end{vmatrix}=1.80065\cdot 10^{-6},$$

$$\Delta_1 = \begin{vmatrix} 0.110896 & 0.069553 & 0.073201 \\ 0.119432 & 0.077453 & 0.078628 \\ 0.126482 & 0.078628 & 0.087529 \end{vmatrix} = 1.12476 \cdot 10^{-6}$$

$$\Delta_2 = \begin{vmatrix} 0.066347 & 0.110896 & 0.073201 \\ 0.069553 & 0.119432 & 0.078628 \\ 0.073201 & 0.126482 & 0.087529 \end{vmatrix} = 9.08189 \cdot 10^{-7},$$

$$\Delta_3 = \begin{vmatrix} 0.066347 & 0.069553 & 0.110896 \\ 0.069553 & 0.077453 & 0.119432 \\ 0.073201 & 0.078628 & 0.126482 \end{vmatrix} = 8.45526 \cdot 10^{-7}$$

then

$$c_1 = \frac{\Delta_1}{\Delta} = 0.624641, \quad c_2 = \frac{\Delta_2}{\Delta} = 0.504369,$$

$$c_3 = \frac{\Delta_2}{\Delta} = 0.469568. \tag{2.149}$$

Thus, taking into account (2.149), formula (2.143) takes the form

$$y_m = 0.624641 \cdot x_1 + 0.504369 \cdot x_2 + 0.469568 \cdot x_3. \tag{2.150}$$

Taking into account the use of logarithms in calculations, formula (2.143) can be written as follows:

$$Q_m = q_1^{0.624641} \cdot q_2^{0.504369} \cdot q_3^{0.469568}. \tag{2.151}$$

Fig. 2.7 presents graphs of real values of power quality and calculated according to the mathematical model (2.151).

Comparison of the graphs shown in Fig. 2.7 shows their rather good similarity. Moreover, the calculated pair correlation coefficient is

$$r_{QQ_m} = 0.923. \tag{2.152}$$

According to the Chaddock scale, since the inequality holds

$$0.9 < r_{QQ_m} < 0.99,$$

then there is a "very high" relationship between the variables.

Thus, formula (2.151) represents a three-factor mathematical model for the quality of electricity.

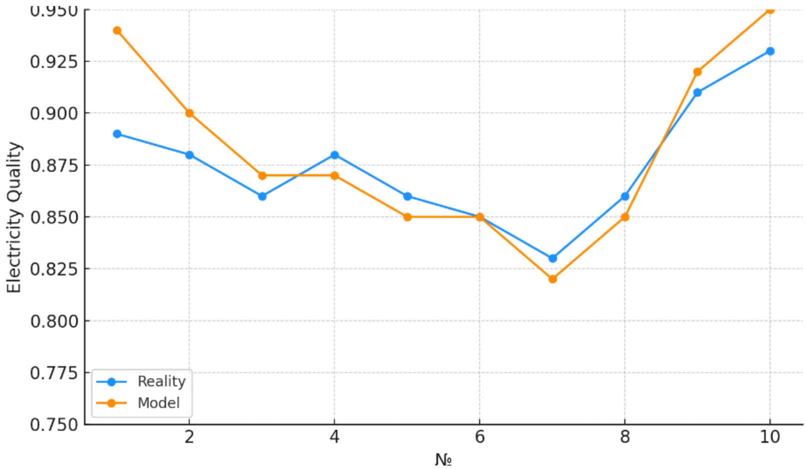

Figure 2.7. Graphs of real power quality values and those calculated by the model (2.151)

Using formula (2.141), it is possible to write the relative error of the overall quality of electricity depending on the relative errors of the qualities of electricity of the three-factor mathematical model of the quality of electricity

$$\delta Q_m = 0.624641 \cdot \delta q_1 + 0.504369 \cdot \delta q_2 + 0.469568 \cdot \delta q_3,$$

where $\delta q_1 -$ relative voltage quality error U,

$\delta q_2 -$ relative error of frequency quality ω,

$\delta q_3 -$ relative error in the quality of the shape of the electric current curve χ.

In the conditions of market relations, electricity should be considered as a commodity that must meet a certain quality. This requires paying special attention to the quality of electricity. In turn, the quality of electricity is determined by the qualities of the electricity of the components that make it up. The construction of a multifactor mathematical model of electricity quality in the form of a static multiplicative model made it possible to study the influence of the components of individual components on electricity on the overall quality. And, ultimately, to indicate ways to improve the quality of electricity, relying on certain indicators of the quality of the components that make up the overall quality of electricity. In addition, the synthesized multifactor mathematical model of electricity quality made it possible to estimate the relative error in determining the overall quality of electricity as a weighted sum of the relative errors of the qualities of the electricity of its components.

2.3 Value-based power quality studies

Conducted research on the quality of electricity from the point of view of a multifactor model, provided grounds for the possibility of representing the integrated parametric parts that make it up.

In particular, a multifactor mathematical model of electricity quality can be represented in a multiplicative form

$$Q = q_1^{c_1} \cdot q_2^{c_2} \cdot \ldots \cdot q_I^{c_I},$$

or in a rolled-up form

$$Q = \prod_{i=1}^{I} q_i^{c_i}, \tag{2.153}$$

where Q – overall model quality, q_i, $(i = 1, 2, \ldots, I)$ – partial quality, c_i, $(i = 1, 2, \ldots, I)$ – parameters, I – number of partial qualities.

The mathematical model (2.153) is considered given if the parameters found by the identification method are determined. With a known mathematical model (2.153), it is advisable to solve the cost-objective problem of finding such values of partial qualities that would minimize the cost of the overall quality of electricity (2.153). Математично це можна записати у вимогі мінімізування лінійної комбінації вартостей частинних якостей

$$S = s_1 \cdot q_1 + s_2 \cdot q_2 + \ldots + s_I \cdot q_I \to \min_{q_1, q_2, \ldots, q_I},$$

or in a rolled-up form

$$S = \sum_{l=1}^{I} s_l \cdot q_l \to \min_{q_1, q_2, \ldots, q_I}, \tag{2.154}$$

where s_i – the cost of a partial quality, provided that the general quality is maintained (153).

Thus, the optimization problem is to find the minimum of the function (154) under the condition of preserving the value of the total quality (153), i.e.

$$S = \sum_{l=1}^{I} s_l \cdot q_l \to \min_{q_1, q_2, \dots, q_I},$$

$$Q = \prod_{i=1}^{I} q_i^{c_i} = \text{const}.$$

To solve this problem, we will use the Lagrange method. To do this, the Lagrange function is constructed using formulas (2.153) and (2.154)

$$L(q_1, q_2, \dots, q_I, \lambda) = s_1 \cdot q_1 + s_2 \cdot q_2 + \dots + s_I \cdot q_I + \lambda \left(Q - q_1^{c_1} \cdot q_2^{c_2} \cdot \dots \cdot q_I^{c_I} \right), \quad (2.155)$$

where λ – parameter.

Next, we calculate the partial derivatives of the Lagrange function (155) with respect to the variables, including the parameter λ, and according to the necessary condition for the existence of an extremum, we set them equal to zero.

$$\frac{\partial L}{\partial q_1} = s_1 - \lambda \cdot c_1 \cdot q_1^{c_1} \cdot q_2^{c_2} \cdot \dots \cdot q_I^{c_I} \cdot q_1^{-1} = 0,$$

$$\frac{\partial L}{\partial q_2} = s_2 - \lambda \cdot c_2 \cdot q_1^{c_1} \cdot q_2^{c_2} \cdot \dots \cdot q_I^{c_I} \cdot q_2^{-1} = 0, \quad (2.156)$$

$$\frac{\partial L}{\partial q_I} = s_I - \lambda \cdot c_I \cdot q_1^{c_1} \cdot q_2^{c_2} \cdot \dots \cdot q_I^{c_I} \cdot q_I^{-1} = 0,$$

$$\frac{\partial L}{\partial \lambda} = Q - q_1^{c_1} \cdot q_2^{c_2} \cdot \dots \cdot q_I^{c_I} = 0$$

As a result, we obtain a system $I+1$ of equations with $I+1$ unknowns (2.156) which we represent in the form

$$\begin{cases} s_1 = \lambda \cdot c_1 \cdot q_1^{c_1} \cdot q_2^{c_2} \cdot \ldots \cdot q_I^{c_I} \cdot q_1^{-1} \\ s_2 = \lambda \cdot c_2 \cdot q_1^{c_1} \cdot q_2^{c_2} \cdot \ldots \cdot q_I^{c_I} \cdot q_2^{-1} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ s_I = \lambda \cdot c_I \cdot q_1^{c_1} \cdot q_2^{c_2} \cdot \ldots \cdot q_I^{c_I} \cdot q_I^{-1} \\ Q = q_1^{c_1} \cdot q_2^{c_2} \cdot \ldots \cdot q_I^{c_I} \end{cases} \quad (2.157)$$

The system of equations (2.157) is nonlinear, so we will use an artificial method to solve it. To do this, we will divide all equations, except the last one, by the first equation

$$\frac{s_2}{s_1} = \frac{c_2}{c_1}\frac{q_1}{q_2},$$

$$\frac{s_3}{s_1} = \frac{c_3}{c_1}\frac{q_1}{q_3}, \quad (2.158)$$

$$\frac{s_I}{s_1} = \frac{c_I}{c_1}\frac{q_1}{q_I}.$$

Next, we solve each of the equations of the system (2.158) with respect to the unknown variable q_1

$$q_2 = \frac{c_2}{c_1}\frac{s_1}{s_2}q_1,$$

$$q_3 = \frac{c_3}{c_1}\frac{s_1}{s_3}q_1, \quad (2.159)$$

$$q_I = \frac{c_I}{c_1}\frac{s_1}{s_I}q_1.$$

Let us substitute the variables from system (2.159) into the last equation of system (2.157)

$$Q = q_1^{c_1} \cdot \left(\frac{c_2}{c_1}\frac{s_1}{s_2}q_1\right)^{c_2} \cdot \left(\frac{c_3}{c_1}\frac{s_1}{s_3}q_1\right)^{c_3} \cdot \ldots \cdot \left(\frac{c_I}{c_1}\frac{s_1}{s_I}q_1\right)^{c_I}. \quad (2.160)$$

Equation (2.160) is an equation of $c_1 + c_2 + ... + c_I$ of the degree of the unknown q_1. Indeed,

$$Q = q_1^{c_1+c_2+c_3+...+c_I} \cdot \left(\frac{c_2}{c_1}\frac{s_1}{s_2}\right)^{c_2} \cdot \left(\frac{c_3}{c_1}\frac{s_1}{s_3}\right)^{c_3} \cdot ... \cdot \left(\frac{c_I}{c_1}\frac{s_1}{s_I}\right)^{c_I} . \quad (2.161)$$

We solve equation (2.161) with respect to the unknown q_1

$$Q = q_1^{c_1+c_2+c_3+...+c_I} \cdot \left(\frac{s_1}{c_1}\right)^{c_2+c_3+...+c_I} \left(\frac{c_2}{s_2}\right)^{c_2} \cdot \left(\frac{c_3}{s_3}\right)^{c_3} \cdot ... \cdot \left(\frac{c_I}{s_I}\right)^{c_I},$$

$$Q^{\frac{1}{c_1+c_2+c_3+...+c_I}} = q_1 \cdot \left[\left(\frac{s_1}{c_1}\right)^{c_2+c_3+...+c_I} \left(\frac{c_2}{s_2}\right)^{c_2} \cdot \left(\frac{c_3}{s_3}\right)^{c_3} \cdot ... \cdot \left(\frac{c_I}{s_I}\right)^{c_I}\right]^{\frac{1}{c_1+c_2+c_3+...+c_I}} \quad (2.162)$$

$$q_1 = Q^{\frac{1}{c_1+c_2+c_3+...+c_I}} \cdot \left[\left(\frac{s_1}{c_1}\right)^{c_2+c_3+...+c_I} \left(\frac{c_2}{s_2}\right)^{c_2} \cdot \left(\frac{c_3}{s_3}\right)^{c_3} \cdot ... \cdot \left(\frac{c_I}{s_I}\right)^{c_I}\right]^{\frac{-1}{c_1+c_2+c_3+...+c_I}},$$

$$q_1 = \left[Q^{-1} \cdot \left(\frac{s_1}{c_1}\right)^{c_2+c_3+...+c_I} \cdot \left(\frac{c_2}{s_2}\right)^{c_2} \cdot \left(\frac{c_3}{s_3}\right)^{c_3} \cdot ... \cdot \left(\frac{c_I}{s_I}\right)^{c_I}\right]^{\frac{-1}{c_1+c_2+c_3+...+c_I}}$$

In a reduced form, formula (2.162) can be written as

$$q_{1,opt} = \left[Q^{-1} \cdot \left(\frac{s_1}{c_1}\right)^{\sum_{i=2}^{I} c_i} \cdot \prod_{k=2}^{I}\left(\frac{c_k}{s_k}\right)^{c_k}\right]^{\frac{-1}{\sum_{i=1}^{I} c_i}} \quad (2.163)$$

Taking into account (2.163), formulas (2.159) allow us to write the values of other unknown variables

$$q_{2,opt} = \frac{c_2}{c_1}\frac{s_1}{s_2}\left[Q^{-1}\cdot\left(\frac{s_1}{c_1}\right)^{\sum_{i=2}^{I}c_i}\cdot\prod_{k=2}^{I}\left(\frac{c_k}{s_k}\right)^{c_k}\right]^{\frac{-1}{\sum_{i=1}^{I}c_i}},$$

$$q_{3,opt} = \frac{c_3}{c_1}\frac{s_1}{s_3}\left[Q^{-1}\cdot\left(\frac{s_1}{c_1}\right)^{\sum_{i=2}^{I}c_i}\cdot\prod_{k=2}^{I}\left(\frac{c_k}{s_k}\right)^{c_k}\right]^{\frac{-1}{\sum_{i=1}^{I}c_i}} \quad (2.164)$$

$$q_{I,opt} = \frac{c_I}{c_1}\frac{s_1}{s_I}\left[Q^{-1}\cdot\left(\frac{s_1}{c_1}\right)^{\sum_{i=2}^{I}c_i}\cdot\prod_{k=2}^{I}\left(\frac{c_k}{s_k}\right)^{c_k}\right]^{\frac{-1}{\sum_{i=1}^{I}c_i}}$$

In general form, formulas (2.163), (2.164) have the form

$$q_{k,opt} = \frac{c_k}{c_1}\frac{s_1}{s_k}\left[Q^{-1}\cdot\left(\frac{s_1}{c_1}\right)^{\sum_{i=2}^{I}c_i}\cdot\prod_{l=2}^{I}\left(\frac{c_l}{s_l}\right)^{c_l}\right]^{\frac{-1}{\sum_{i=1}^{I}c_i}} \qquad (k = 1, 2, ..., I) \quad (2.165)$$

According to (2.165), the minimum cost of the total quality of electricity is written as

$$S_{\min} = \sum_{k=1}^{I} s_k \cdot q_{k,opt},$$

$$S_{\min} = \frac{s_1}{c_1}\frac{Q^{\frac{1}{\sum_{i=1}^{I}c_i}}\sum_{k=1}^{I}c_k}{\left(\frac{s_1}{c_1}\right)^{\frac{\sum_{i=2}^{I}c_i}{\sum_{i=1}^{I}c_i}}\cdot\prod_{l=2}^{I}\left(\frac{c_l}{s_l}\right)^{\frac{c_l}{\sum_{i=1}^{I}c_i}}}. \quad (2.166)$$

As an example, let's consider the optimization of a two-factor model for electricity quality. In this case, the two-factor model for electricity quality has the form

$$Q = q_1^{c_1} q_2^{c_2}, \qquad (2.167)$$

where Q – electricity quality, q_1, q_2 – partial qualities of electricity, c_1, c_2 – parameters.

The values of the parameters included in the formula (2.167) are found by identification according to the given statistical material. Table 1 shows the corresponding statistical material.

In order to use the method of least squares in linear form to find the parameters in the model (2.167), we will apply the logarithm opera

$$\ln Q = c_1 \ln q_1 + c_2 \ln q_2,$$

or $y = c_1 x_1 + c_2 x_2$, (2.168)

where $y = \ln Q$, $x_1 = \ln q_1$, $x_2 = \ln q_2$.

Table 5

Input parameters for the Regression subroutine in Microsoft Excel

№	Q	q_1	q_2	$y = \ln Q$	$x_1 = \ln q_1$	$x_2 = \ln q_2$	Q_m	S
1	0.9	0.95	0.96	-0.10536	-0.05129	-0.04082	0.93	1.238
2	0.89	0.92	0.93	-0.11653	-0.08338	-0.07257	0.88	1.199
3	0.85	0.91	0.93	-0.16252	-0.09431	-0.07257	0.87	1.189
4	0.88	0.93	0.94	-0.12783	-0.07257	-0.06188	0.89	1.212
5	0.84	0.91	0.89	-0.17435	-0.09431	-0.11653	0.84	1.177
6	0.85	0.92	0.9	-0.16252	-0.08338	-0.10536	0.86	1.19
7	0.83	0.9	0.89	-0.18633	-0.10536	-0.11653	0.83	1.167
8	0.87	0.89	0.89	-0.13926	-0.11653	-0.11653	0.83	1.157
9	0.91	0.96	0.92	-0.09431	-0.04082	-0.08338	0.91	1.236
10	0.93	0.98	0.95	-0.07257	-0.0202	-0.05129	0.95	1.265

To find the values of the parameters in formula (2.168), we will use the subroutine "Regression" in Microsoft Excel according to the data in table 2.1.

As a result, we will obtain the regression equation

$$y = 0.9378 \cdot x_1 + 0.6998 \cdot x_2. \qquad (2.169)$$

In this case, the coefficient of determination was

$$R^2 = 0.974\,. \tag{2.170}$$

In turn, the multiple correlation coefficient is

$$R = 0.987\,. \tag{2.171}$$

Since according to (2.171) the condition is satisfied

$$0.9 \le 0.987 \le 1\,,$$

then according to the Chaddock scale [39] there is a "very high" relationship between the variables.

In real terms, according to (2.167), the two-factor model for electricity quality has the form

$$Q_m = q_1^{0.9378} \cdot q_2^{0.6998}\,. \tag{2.172}$$

Fig. 2.8 presents graphs of real values and a two-factor model for electricity quality.

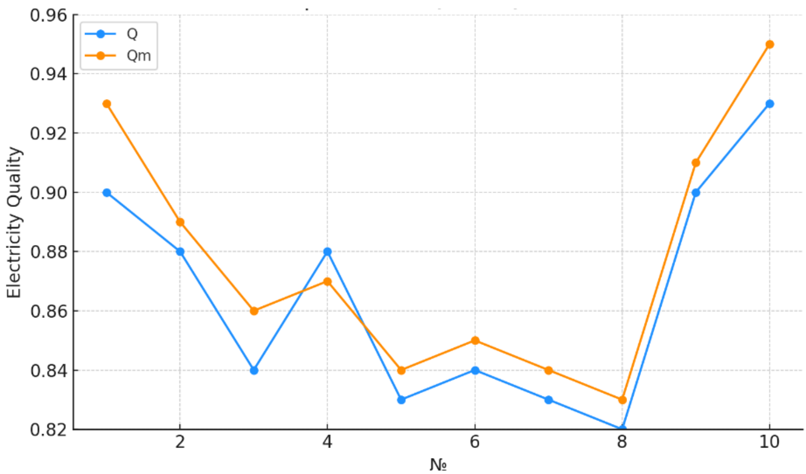

Figure 2.8. Power quality graphs

Analysis of the graphs confirms the convergence of the real values of electricity quality and the values calculated using the two-factor model of electricity quality.

The function that determines the cost of the overall quality of electricity relative to the costs of the individual qualities of electricity has the form

$$S = s_1 q_1 + s_2 q_2, \quad (2.173)$$

where S – cost of overall electricity quality, s_1, s_2 – specific values of partial qualities of electricity.

The task of optimizing a two-factor model for electricity quality is to minimize the cost of the overall electricity quality given a given value of the overall electricity quality (2.167), i.e.

$$S = s_1 q_1 + s_2 q_2 \to \min_{q_1, q_2},$$

$$Q = q_1^{c_1} q_2^{c_2} = \text{const} .$$

Using the second condition, we find the value of the second partial quality of electricity

$$q_2 = \left(\frac{Q}{q_1^{c_1}} \right)^{\frac{1}{c_2}} . \quad (2.174)$$

Then the overall quality of electricity will be defined as a function of one variable, namely, the first partial quality of electricity

$$S = s_1 q_1 + s_2 \left(\frac{Q}{q_1^{c_1}} \right)^{\frac{1}{c_2}},$$

or

$$S(q_1) = s_1 q_1 + s_2 Q^{\frac{1}{c_2}} q_1^{-\frac{c_1}{c_2}} . \quad (2.175)$$

It is advisable to investigate the dependence of the total quality of electricity q_1 (2.175) on the value of the partial quality of electricity at given values of the parameters. The values of the parameters c_1 and c_2 are determined by the results of the identification of the mathematical model (2.153), i.e.

$$c_1 = 0.9378, \ c_2 = 0.6998. \tag{2.176}$$

In turn, we will accept conditionally

$$Q = 0.321, \ s_1 = 1, \ s_2 = 0.3. \tag{2.177}$$

Then, according to (2.176) and (2.177), formula (2.175) will take the form

$$S(q_1) = q_1 + 0.089 q_1^{-1.34}. \tag{2.178}$$

Fig. 2.9 shows the graph of function (2.178).

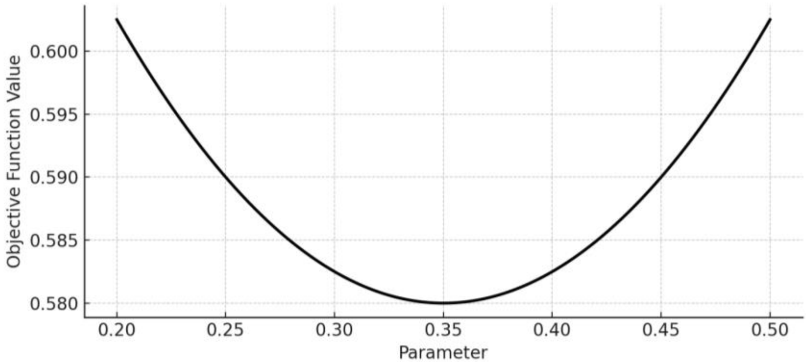

Figure 2.9. Graph of the dependence of the cost of total electricity quality on the value of partial electricity quality

Analysis of the graph shown in Fig. 2.9 shows that the dependence of the cost of the total quality of electricity for the selected parameters on the value of the partial quality of electricity reaches a minimum value at some optimal value of this parameter. Analytically we find that

$$q_{1,opt} = 0.339, \ S_{\min} = S(q_{1,opt}) = 0.591. \tag{2.179}$$

Let's solve the problem of finding the minimum value of the total quality of electricity analytically.

To find the minimum value of the total quality of electricity (2.175), we will use the necessary condition for the existence of an extremum of a function of one variable. According to this condition, it is necessary that at the point of the extremum the derivative of the function is equal to zero. We find the derivative of the function (2.175)

$$\frac{dS}{dq_1} = s_1 + s_2 Q^{\frac{1}{c_2}} \left(-\frac{c_1}{c_2} \right) q_1^{-\frac{c_1}{c_2}-1} . \tag{2.180}$$

We set the derivative of function (2.179) to zero and solve the resulting equation with respect to q_1

$$s_1 + s_2 Q^{\frac{1}{c_2}} \left(-\frac{c_1}{c_2} \right) q_1^{-\frac{c_1}{c_2}-1} = 0 ,$$

$$s_1 = s_2 \frac{c_1}{c_2} Q^{\frac{1}{c_2}} q_1^{-\frac{c_1}{c_2}-1} ,$$

$$s_1 = s_2 \frac{c_1}{c_2} Q^{\frac{1}{c_2}} q_1^{-\frac{c_1+c_2}{c_2}} \tag{2.181}$$

$$q_1^{\frac{c_1+c_2}{c_2}} = \frac{s_2}{s_1} \frac{c_1}{c_2} Q^{\frac{1}{c_2}} ,$$

$$q_1 = \left(\frac{s_2}{s_1} \frac{c_1}{c_2} Q^{\frac{1}{c_2}} \right)^{\frac{c_2}{c_1+c_2}}$$

To determine the nature of the found critical point (2.181), we use the second-order derivative. To do this, we find the derivative of the derivative (2.180)

$$\frac{d^2 S}{dq_1^2} = s_2 Q^{\frac{1}{c_2}} \left(-\frac{c_1}{c_2} \right) \left(-\frac{c_1}{c_2} - 1 \right) q_1^{-\frac{c_1}{c_2}-2} . \tag{2.182}$$

We substitute the value of the critical point (2.181) into (2.182)

$$\frac{d^2}{dq_1^2} S \left[q_1 = \left(\frac{s_2}{s_1} \frac{c_1}{c_2} Q^{\frac{1}{c_2}} \right)^{\frac{c_2}{c_1+c_2}} \right] = s_2 Q^{\frac{1}{c_2}} \left(-\frac{c_1}{c_2} \right) \left(-\frac{c_1}{c_2} - 1 \right) \left(\left(\frac{s_2}{s_1} \frac{c_1}{c_2} Q^{\frac{1}{c_2}} \right)^{\frac{c_2}{c_1+c_2}} \right)^{-\frac{c_1}{c_2}-2}$$

$$\frac{d^2}{dq_1^2}S\left[q_1=\left(\frac{s_2}{s_1}\frac{c_1}{c_2}Q^{\frac{1}{c_2}}\right)^{\frac{c_2}{c_1+c_2}}\right]=s_2Q^{\frac{1}{c_2}}\left(\frac{c_1}{c_2}\right)\left(\frac{c_1}{c_2}+1\right)\left(\left(\frac{s_2}{s_1}\frac{c_1}{c_2}Q^{\frac{1}{c_2}}\right)^{\frac{c_2}{c_1+c_2}}\right)^{-\frac{c_1}{c_2}-2}>0 \tag{2.183}$$

Since the value of the second-order derivative at the critical point is positive, the function (2.178) has a minimum at this point. Thus, we have

$$q_{1,opt}=\left(\frac{s_2}{s_1}\frac{c_1}{c_2}Q^{\frac{1}{c_2}}\right)^{\frac{c_2}{c_1+c_2}}. \tag{2.184}$$

Using formula (2.177), we find the optimal value of the second partial quality of electricity

$$q_{2,opt}=Q^{\frac{1}{c_2}}q_{1,opt}^{-\frac{c_1}{c_2}}$$

$$q_{2,opt}=Q^{\frac{1}{c_2}}\left[\left(\frac{s_2}{s_1}\frac{c_1}{c_2}Q^{\frac{1}{c_2}}\right)^{\frac{c_2}{c_1+c_2}}\right]^{-\frac{c_1}{c_2}} \tag{2.185}$$

$$q_{2,opt}=Q^{\frac{1}{c_2}}\left(\frac{s_2}{s_1}\frac{c_1}{c_2}Q^{\frac{1}{c_2}}\right)^{-\frac{c_1}{c_1+c_2}}$$

Using (2.184) and (2.185), we find the minimum value of the total electricity quality cost

$$S_{\min}=s_1\left(\frac{s_2}{s_1}\frac{c_1}{c_2}Q^{\frac{1}{c_2}}\right)^{\frac{c_2}{c_1+c_2}}+s_2Q^{\frac{1}{c_2}}\left(\frac{s_2}{s_1}\frac{c_1}{c_2}Q^{\frac{1}{c_2}}\right)^{-\frac{c_1}{c_1+c_2}} \tag{2.186}$$

To verify the obtained results (2.184), (2.185) and (2.186), we substitute the values of the parameters (2.176) and (2.177) into them

$$q_{1,opt} = \left(\frac{0.3}{1} \cdot \frac{0.9378}{0.6998} \cdot 0.321^{\frac{1}{0.6998}} \right)^{\frac{0.6998}{0.9378+0.6998}} = 0.3385 \quad (2.187)$$

$$q_{2,opt} = 0.321^{\frac{1}{0.6998}} \left(\frac{0.3}{1} \cdot \frac{0.9378}{0.6998} \cdot 0.321^{\frac{1}{0.6998}} \right)^{-\frac{0.9378}{0.9378+0.6998}} = 0.8419 \quad (2.188)$$

$$S_{\min} = S(q_{1,opt}, q_{2,opt}) = 1 \cdot 0.3385 + 0.3 \cdot 0.8419 = 0.5911 \quad (2.189)$$

The results (2.187) and (2.188) coincide with (2.179), which confirms the correctness of the calculations.

The last column of table 2.5 shows the results of calculating the values of the total quality of electricity according to formula (2.173). The minimum value calculated is

$$S = 1.157 . \quad (2.190)$$

Thus, a comparison of the calculated values of the overall quality of electricity, according to formulas (2.187) and (2.188), indicates that by minimizing the cost of the overall quality of electricity, the

$$\frac{S}{S_{\min}} = \frac{1.157}{0.5911} = 1.957 \approx 2 \text{ times.} \quad (2.191)$$

It should be emphasized that the partial qualities of electricity must be within the range $[0,1]$, i.e. satisfy the constraint

$$0 \le q_i \le 1 . \quad (2.192)$$

Thus, according to (2.184) it must hold

$$q_{1,opt} = \left(\frac{s_2}{s_1} \frac{c_1}{c_2} Q^{\frac{1}{c_2}} \right)^{\frac{c_2}{c_1+c_2}} \le 1 \quad (2.193)$$

Condition (2.193) imposes a bound on the ratio of specific values. Indeed, for the first partial quality $q_{1,opt}$ we have

$$\left(\frac{s_2}{s_1}\frac{c_1}{c_2}Q^{\frac{1}{c_2}}\right)^{\frac{c_2}{c_1+c_2}} \le 1$$

$$\frac{s_2}{s_1}\frac{c_1}{c_2}Q^{\frac{1}{c_2}} \le 1 \qquad (2.194)$$

$$\frac{s_2}{s_1} \le \frac{c_2}{c_1}Q^{-\frac{1}{c_2}}$$

Similarly, for the second partial quality $q_{2,opt}$

$$q_{2,opt} = Q^{\frac{1}{c_2}}\left(\frac{s_2}{s_1}\frac{c_1}{c_2}Q^{\frac{1}{c_2}}\right)^{-\frac{c_1}{c_1+c_2}} \le 1, \quad \left(\frac{s_2}{s_1}\frac{c_1}{c_2}Q^{\frac{1}{c_2}}\right)^{-\frac{c_1}{c_1+c_2}} \le Q^{\frac{-1}{c_2}},$$

$$\frac{s_2}{s_1}\frac{c_1}{c_2} \le Q^{\frac{1}{c_2}\frac{c_1}{c_1+c_2}} \cdot Q^{\frac{-1}{c_2}}, \quad \frac{s_2}{s_1}\frac{c_1}{c_2} \le Q^{\frac{1}{c_2}\frac{c_1}{c_1+c_2}+\frac{-1}{c_2}}, \qquad (2.195)$$

$$\frac{s_2}{s_1}\frac{c_1}{c_2} \le Q^{\frac{1}{c_2}\left(\frac{c_1}{c_1+c_2}-1\right)}, \quad \frac{s_2}{s_1}\frac{c_1}{c_2} \le Q^{\frac{1}{c_2}\left(\frac{c_1-c_1-c_2}{c_1+c_2}\right)},$$

$$\frac{s_2}{s_1}\frac{c_1}{c_2} \le Q^{\frac{-1}{c_1+c_2}}, \quad \frac{s_2}{s_1} \le \frac{c_2}{c_1}Q^{\frac{-1}{c_1+c_2}}.$$

As the results of the calculations (2.187) and (2.188) showed, the conditions (2.194) and (2.193) are fulfilled. Indeed, there is

$$\frac{s_2}{s_1} = \frac{0.3}{1} = 0.3, \quad \frac{c_2}{c_1}Q^{-\frac{1}{c_2}} = 3.778, \quad \frac{c_2}{c_1}Q^{\frac{-1}{c_1+c_2}} = 1.492,$$

which confirms the fulfillment of conditions (2.194) and (2.195).

Of course, if conditions (2.194) or (2.195) were not fulfilled, the ratio of specific values would have to be changed.

Thus, the conducted complex of scientific research, the main provisions of which are set out in this monograph, gives grounds to assert the expediency of processing the acquired scientific achievements in the further process of searching and developing logistics and circuit-technical solutions, regarding management actions to maintain the current indicators of electricity quality at the level of the relevant standards of Ukraine and the European Union.

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MONOGRAPH

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